1
Fundamental Interactions

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1 Cosmic Rays

Exercise 1

High energy primary cosmic rays consist of protons, α particles and heavy nuclei. Only 1% of primary particles are electrons. Does this mean that the planet Earth will be electrically charged in the course of time because of the continuous bombardment by predominantly positive charged primary cosmic rays? How much positive charge excess could have been accumulated during the period of existence of our planet if this were true? Try to explain why the Earth is not significantly positively charged. Consider that the rate of primary particles can be estimated as $\Phi_0 \sim 0.2$ (cm^2 sec sr)$^{-1}$. Consider that Earth's radius is $R = 6370$ km, and its age is $T = 4.5 \times 10^9$ yrs.

Exercise 2

Consider a cosmic ray detector that floats on a balloon at an altitude of 35 km over sea level. The detector is a scintillator with perfect detection probability and a surface area of 75x75 cm.

- How many particles with energy larger than 10^{14} eV will be detected per day? Assume that the cosmic ray spectrum is given by the following with enough accuracy

$$\Phi(E) = \frac{dN}{dE\, dA\, d\Omega\, dt} = 1.1 \times 10^{-18}\, \mathrm{eV}^{-1}\, \mathrm{cm}^{-2}\, \mathrm{sr\, s} \left(\frac{E}{10^{14}\, \mathrm{eV}} \right)^{-2.7}$$

- How many cosmic rays with energy in excess of 1 GeV hit the Earth each second. Ignore the deflection due to the Earths magnetic field. What is the average mass collected by the Earth over a year, assuming most of the cosmic radiation is made of protons.

Exercise 3

Show that the gyroradius of a particle of mass m that travels at velocity v in a magnetic field B is given by

$$r_L = \frac{p_\perp}{ZeB}, \tag{1}$$

with $p_\perp$ the component of particle momentum perpendicular to the magnetic field, Ze the electric charge of the particle. Show that this expression can be numerically expressed as

$$r_L \sim \frac{3.3 \times 10^4}{Z} \frac{E(GeV)}{B(G)} m, \tag{2}$$

where the magnetic field is expressed in Gauss.

Show that it can be written also as

$$r_L \sim \frac{1.1 \times 10^{-3}}{Z} \frac{E(TeV)}{B(\mu G)} pc. \tag{3}$$

Exercise 4

It is believed that particles with energy larger than 10^{19} eV originate in extragalactic sources. Calculate the gyroradius of one such particle in a homogeneous magnetic field as strong as the typical galactic magnetic field of B = 3 μG. Consider a proton and iron nuclei with an energy of 10^{18} eV and compare with the size of the Galaxy.

Exercise 5

Considering that the Earth magnetic field has an intensity between 25 μT and 65 μT, calculate the maximum energy a muon should have to have a gyroradius equal to the Earth radius. And for a gyroradius equal to the muon production altitude in the atmosphere (about 30 km).

2 Lorentz Invariance

Exercise 1

Consider the LHC proton-proton accelerator with a total energy of $E' = 7$ TeV and $E' = 14$ TeV, respectively. Determine what is the equivalent total LHC energy in the laboratory frame (which is relevant in cosmic ray physics). What is the fastest way to do the calculation ?

Exercise 2

Consider the scattering process $p + p \to p + p$ in the center of mass frame (like in LHC accelerator), where p is a proton. Show that the total energy is determined by s and the scattering angle is determined by t. Express the solid angle interval $d\Omega$ in t.

Exercise 3 - Mandelstam variables

We define the Mandelstam variables for a relativistic (particle) scattering process shown in figure 2 as follows

$$s = (p_1^\mu + p_2^\mu)^2$$
$$t = (p_1^\mu + p_2^\mu)^2$$
$$u = (p_1^\mu + p_4^\mu)^2$$

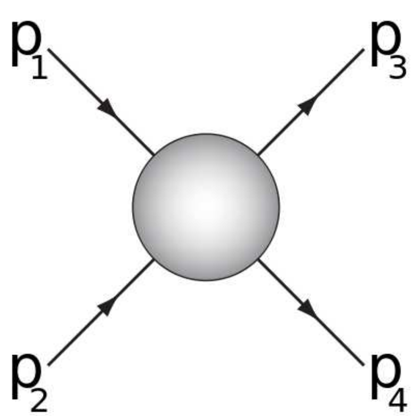

Figure 1: generic scattering process

Show the following useful relation

$$s + t + u = \sum_{i=0}^{4} m_i^2$$

and express the energies E_i, the three-momenta $|p_i|$ and the scattering angle θ in the CM reference system in terms of the Mandelstam variables and the particle masses m_i. Furthermore calculate s, t and u for the case of massless particles if $E_1 = E$ and θ are given (again in CM reference system).

3 Rutherford Scattering

Exercise 1

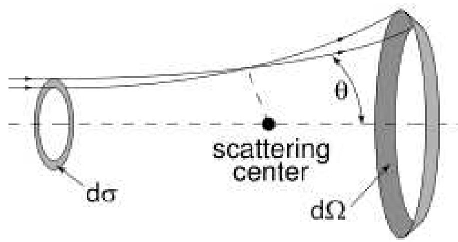

Figure 2: Geometry for differential cross section as a function of impact parameter in spherical coordinates.

Considering that $d\sigma = 2\pi\, b\, db$, find the differential cross section per unit solid angle $\frac{d\sigma}{d\Omega}$ as a function of impact parameter b and scattering angle θ (see Fig. 2).

As long as the incoming particle energy is not too large (as it was in the Rutherford experiment) the trajectory can be solved using classical mechanics. It is possible to show that the impact parameter as a function of scattering angle is

$$b = \left(\frac{Z_1\, Z_2\, e^2}{4\pi\, \epsilon_0\, E} \right) \cot \frac{\theta}{2}, \tag{4}$$

with Z_1 the α particle atomic number, Z_2 that of gold and E the energy of the α particle. So the Rutherford differential cross section is obtained.

Calculate the differential cross section $\frac{d\sigma}{d\Omega}$ at $\theta = 30°$, $60°$ and $90°$ for α particles with energy 7.7 MeV incident on a gold atom ($^{197}_{79}$Au). Consider that in nuclear physics cross sections are expressed in units of barn, b $= 10^{-28}$ m^2.

Exercise 2

Consider α particles with energy 7.7 MeV incident on a gold ($^{197}_{79}$Au) foil of thickness L=1μm. The density of the gold foil is $\rho = 19300$ kg m^{-3}.

If R_i is the rate of incoming α particles (sec^{-1}) and R_s is the rate of scattered particles (sec^{-1}), calculate the fraction of scattered α particles at scattering angles $\theta = 30°$, $60°$ and $90°$ within a band $d\theta = 1°$. We assume that the gold foil is thin enough that only one scattering process per cross occurs (i.e. no multiple scattering). Dimensional analyses are always useful.

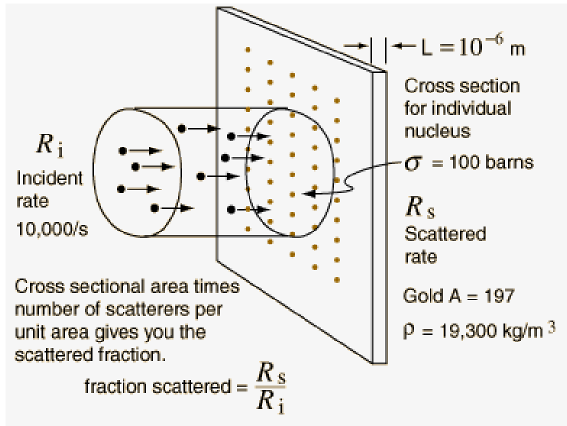

Figure 3: Rutherford Scattering on a thin gold foil

Exercise 3

A head-on collision is when impact parameter b=0, which corresponds to a back-scattered alpha particle.

1. Determine the distance of closest approach to target nucleus in a head-on collision. Consider that this is an elastic scattering process and that at closest approach the α particle goes to a complete stop.

2. Calculate the closest approach of a 7.7 MeV α particle to a gold and Aluminium nuclei ($^{27}_{13}$Al). Consider that the nuclear radius is approximately $r = A^{1/3}\,1.2$ fm.

4 Thomson Scattering

Exercise 1

Consider a gas with a density of N molecules per unit of volume; let n be the number of electrons per molecule. Run a light beam of intensity Φ through the gas, of sufficiently short wavelength that the electrons respond to the beam as if they were free particles. Show that due to scattering by the electrons the intensity of the light beam decreases as a function of distance x traveled through the gas according to the formula

$$\frac{d\Phi}{dx} = -nN\sigma_e\Phi$$

Solve this equation, and show that Φ decays exponentially over a typical path length

$$l = \frac{1}{nN\sigma_e}$$

Evaluate this length for hydrogen (H2) at standard temperature and pressure (T = 273 K and p = 1 atm).

Exercise 2

The warm ionized interstellar medium (ISM) of our Galaxy in the solar neighborhood can be approximated by a disk of half-thickness h≈1 kpc, electron density $N_3 \approx 0.1\text{cm}^{-3}$, and electron temperature T≈104 K. What is the optical depth τ of the ISM to Thomson scattering in the direction normal to the disk?

Note: Optical depth measures the attenuation of the transmitted radiant power in a material. Attenuation can be caused by absorption, but also reflection, scattering, and other physical processes.

%

5 Compton scattering

Exercise 1

Compton scattering is the scattering process between an energetic photon ($E_\gamma \sim m_e c^2$) and an electron, where part of photon energy is transferred to the electron. Because of energy conservation the scattered photon has a correspondingly smaller energy, thus a longer wavelength. This demonstrated the particle characteristics of photons, in contrast to the classic electromagnetic view, which is recovered at lower energy and where the reemitted photon does not change its wavelength.

Consider an incoming photon of initial energy E_γ hitting an electron initially at rest. After the scattering process the photon diverts by an angle θ compared to its initial direction and its energy is E'_γ and the electron has a relativistic total energy of E'_e.

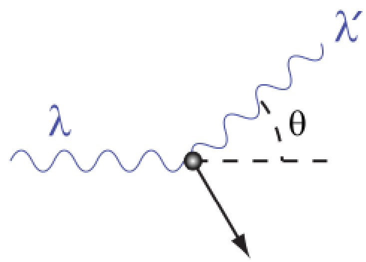

Figure 4: Geometry of Compton Scattering.

Using conservation of energy and momentum show that

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta) \tag{5}$$

Exercise 2

In the same Compton scattering process, show that the final energy of the photon is

$$E'_\gamma = \frac{E_\gamma}{1 + \frac{E_\gamma}{m_e c^2}(1 - \cos\theta)} \tag{6}$$

What is the maximum energy the photon can achieve, given the initial conditions (Compton Edge) ?