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Development of electromagnetic showers - Exercises

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1 Special Relativity

Exercise 1

Prove that $ds = dx^\mu dx_\mu$ is invariant under Lorentz transformation:

$$\Lambda^\mu_\nu = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Exercise 2

Show that one may establish a relation between the delay of the arrival time of a neutrino (with small non negligible mass) and a photon from a distant supernova, and the mass of the neutrino

$$\Delta t = (1\ \text{sec}) \left(\frac{R}{20\,\text{kpc}}\right) \left(\frac{m_\nu}{10\,\text{eV}}\right)^2 \left(\frac{\langle E_\nu \rangle}{10\,\text{MeV}}\right)^{-2}$$

Hint: Assume that neutrinos move at speed $c - \delta c = c(1 - \epsilon)$ with $\epsilon \ll 1$. Use Taylor expansions.

Exercise 3

Demonstrate that it is possible for objects moving towards Earth at an angle, θ to appear to be moving with superluminal speeds. At what value of θ is this effect maximized as a function of the objects speed β?

2 Energy losses

Exercise 1

The energy loss of particles in matter are described by an ionization term (weakly dependent on the particle energy and expressed by Bethe-Bloch formula) and by radiative terms (linearly dependent on the particle energy). This can be parametrized as

$$-\frac{dE}{dX} = \alpha + \beta\, E$$

Derive the expression for the range of a particle with initial energy E_0 and the expression for the minimum energy E_{min} that a particle must have to reach a column depth of X_0.

- Calculate the range for muons in water, knowing that a_μ = 0.259 GeV/mwe and b_μ = 0.364×10^{-3}/mwe.
- Calculate also the range of muons in rock, where a_μ = 0.223 GeV/mwe and b_μ = 0.463×10^{-3}/mwe ($\rho_{rock} = 2.65$ g/cm^3).

Note that mwe corresponds to the column density of 1 meter of water (with volume density of 1 g/cm^3).

3 Electromagnetic Showers

Exercise 1

Verify that the minimum energy a photon must have to create an electron-positron pair in the presence of a stationary nucleus of mass M is

$$2mc^2(1 + m/M),$$

where m is the electron rest mass.

Find the minimum energy needed for pair production in the presence of a proton.

Exercise 2

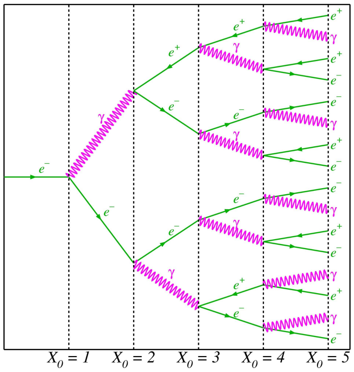

Figure 1: Development of electromagnetic shower according to Heitler model.

The Heitler model of an electromagnetic shower initiated by an energetic gamma ray or electron with energy E_0 is shown in Fig. 1. It is assumed that across each radiation length $X_0 = 37$ g cm^{-2} either a pair production or a bremsstrahlung event occur that multiply the number of particles. It is also assumed that at each stage the two produced particles carry half of the energy of the parent particle.

- What is the number of particles in the shower after a distance X ?
- What is the average particle energy after this distance, relative to the incident particle energy?
- The production of additional particles stops when the particle energy drops below the critical energy (Ec = 81 MeV in air) where ionization losses become dominant. This can be identified as the maximum shower depth. Determine the maximum

atmospheric depth X, defined as a function of the number of radiation lengths X_0 the shower takes to reach its maximum.

- Calculate the number of particles in the shower maximum.
- Give numbers for primary photons with 10^{12} eV and 10^{20} eV. What is the radiation length in air ?).

4 Hadronic Showers

Exercise 1

The Earth's atmosphere is approximately composed by 78% of nitrogen molecules (N_2, ${}^{14}_{7}N$), 22% by oxygen molecules (O_2, ${}^{16}_{8}O$).

- Calculate the nucleon interaction length in the atmosphere, considering that nucleon-air interaction cross section is $\sigma_N^{air} \approx 300$ mb, in the TeV range.
- What is the average height above the sea level where this interaction takes place? Assume isothermal atmospheric profile.

Exercise 2

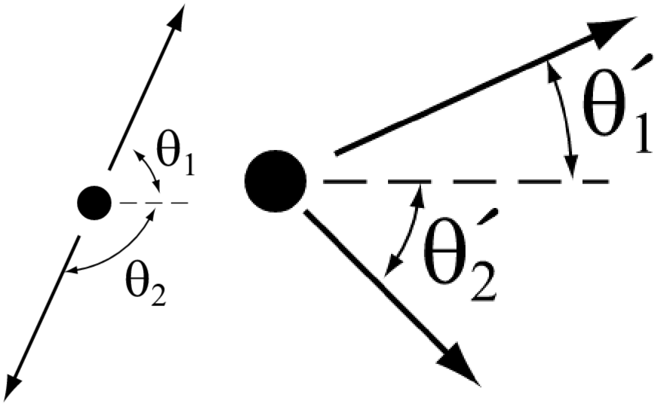

Figure 2: Pion decay graph in the CM reference system (left) and in the LAB frame (right)

In the decay of a charged pion ($m_\pi = 139.570$ MeV) to a muon ($m_\mu = 105.659$ MeV) and a neutrino ($m_\nu \sim 0$ MeV):

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

find the energy carried by the neutrino in the rest frame of the decaying pion.

Calculate the minimum and maximum energies of the neutrino seen by an observer in the laboratory frame in the case of the decay in flight of a pion with momentum 500MeV/c in the lab frame.

Assuming the original pion was traveling along the positive x axis of the laboratory frame, draw the momenta of the outgoing neutrino in these two cases.

Exercise 3

Muons are approximately produced at an atmospheric altitude of 10 km and their mean lifetime is $\tau_\mu = 2.2\,\mu$sec. Calculate the minimum muon energy so than more than 90% of produced muons make it to ground. Assume muons do not lose energy and that they are all produced at 30 km altitude.

5 Neutrino Detection

Exercise 1

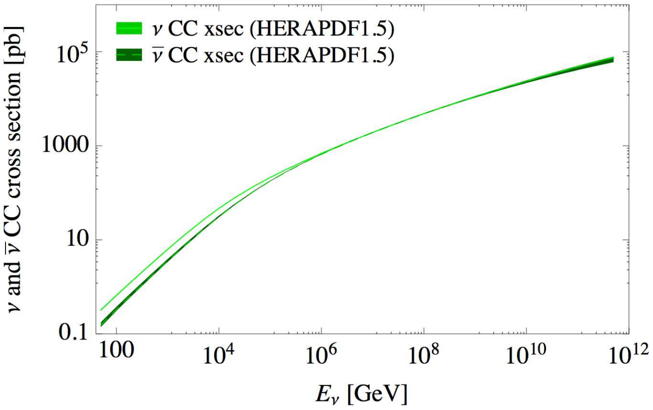

Figure 3: Charged Current interaction cross sections for neutrinos. From Cooper-Sarkar et al. JHEP 08 (2011) 042 (arXiv:1106.3723)

Let's assume we have a muon neutrino intensity of

$$\Phi_\nu(E_\nu) = 1.44 \times 10^{-8}\, E^{-2}\, GeV^{-1}\, cm^{-2}\, sr^{-1}\, sec^{-1}. \tag{1}$$

This flux actually corresponds at the 90% Confidence Level limit on muon neutrino flux from 59 strings of IceCube (arXiv:1311.7048).

An experiment such as IceCube, consists of 1 km^3 instrumented volume buried in the transparent ice in the Antarctic glacier. Muons are detected in the instrumented volume by the Cherenkov light they emit while propagating in the ice.

In general events coming from above (the southern sky) are dominated by the cosmic ray-induced penetrating muons, therefore neutrinos are usually searched only in the upward directions (because muons produced in the atmosphere on the other side of the Earth are absorbed). But at sufficiently high energy neutrino interaction length becomes smaller than the Earth diameter and neutrinos start to be absorbed in the Earth. At such high energies searching for neutrinos from below is not a good idea, so we have to go back to the southern hemisphere and look for events starting inside the detector, in order filter out those events (such as the muons) that have always an entry point in the detector. In any case let's suppose the search is done on half sky.

Calculate how many neutrino-induced muons would be detected by such an instrument in 1 year from the southern sky (half of the whole sky) at the energy of 1 PeV (10^{15} eV, considered as a delta function), from the given neutrino flux. Assume that all neutrinos undergo charged current interactions in the ice.

From Fig. 3 it is possible to see that the neutrino cross section at 1 PeV is approximately 1 nb. Note that the muon intensity is

$$\Phi_\mu(E_\mu) \approx \Phi_\nu(E_\nu) \times P_{\nu\mu}, \tag{2}$$

where $P_{\nu\mu}$ is the probability that neutrinos hit a nucleon in the ice to produce a muon via CC interactions above energy of E_μ and that the muon propagates long enough to be detected at some distance. This is approximately the ratio between the muon range and the neutrino interaction length in ice.