workshop in particle physics

fundamental interactions & electromagnetic showers

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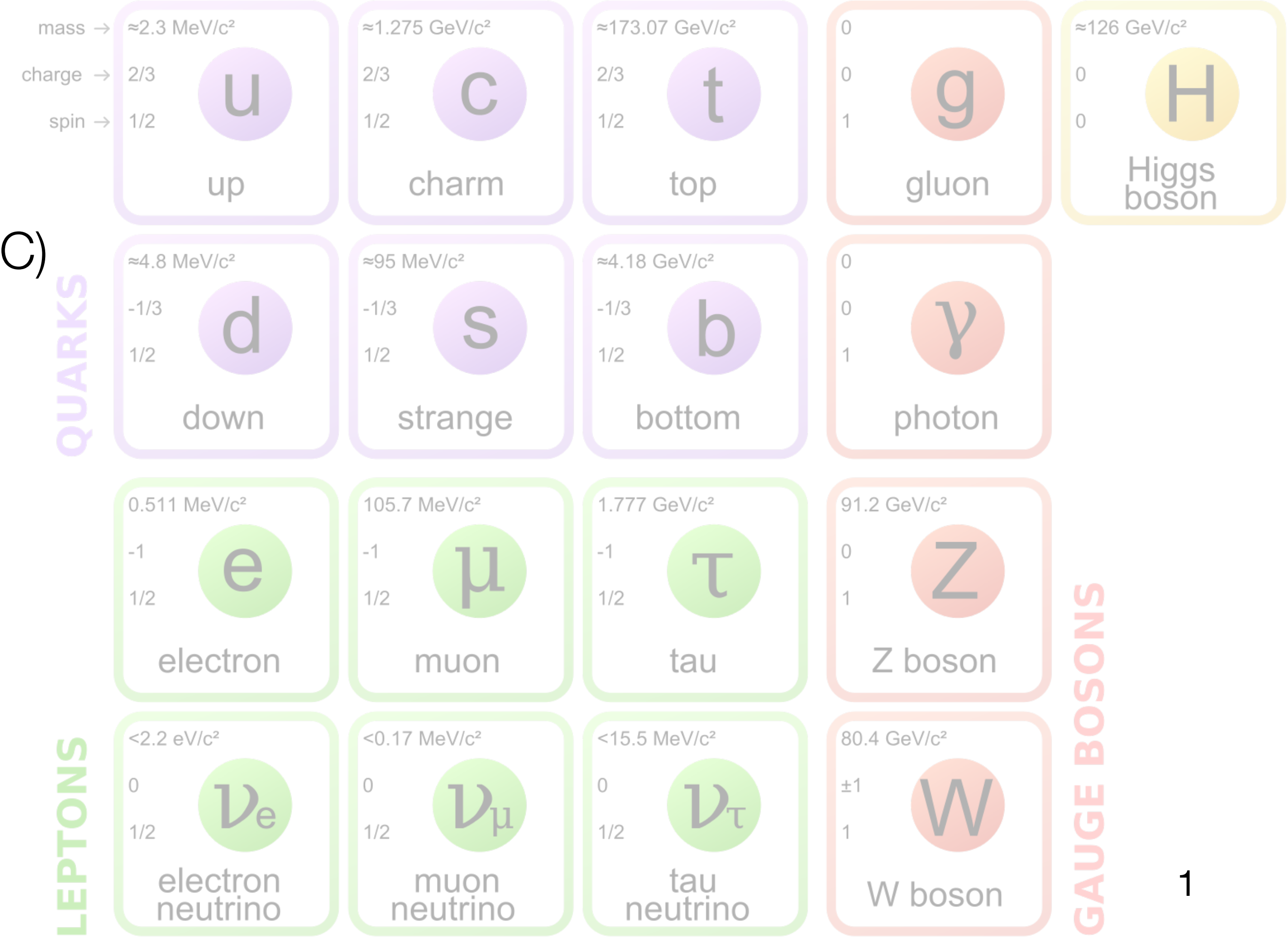
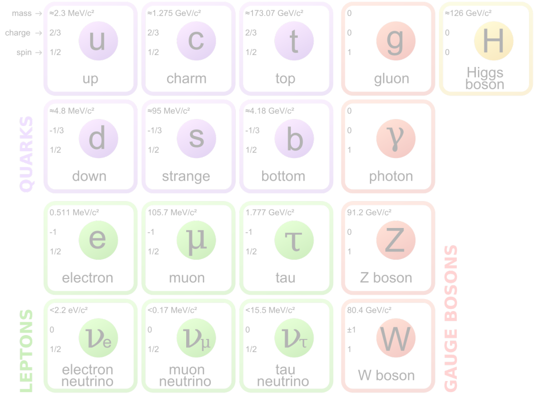


lectures outline

workshop in particle physics



1. fundamental interactions & EM showers

2. development of electromagnetic showers

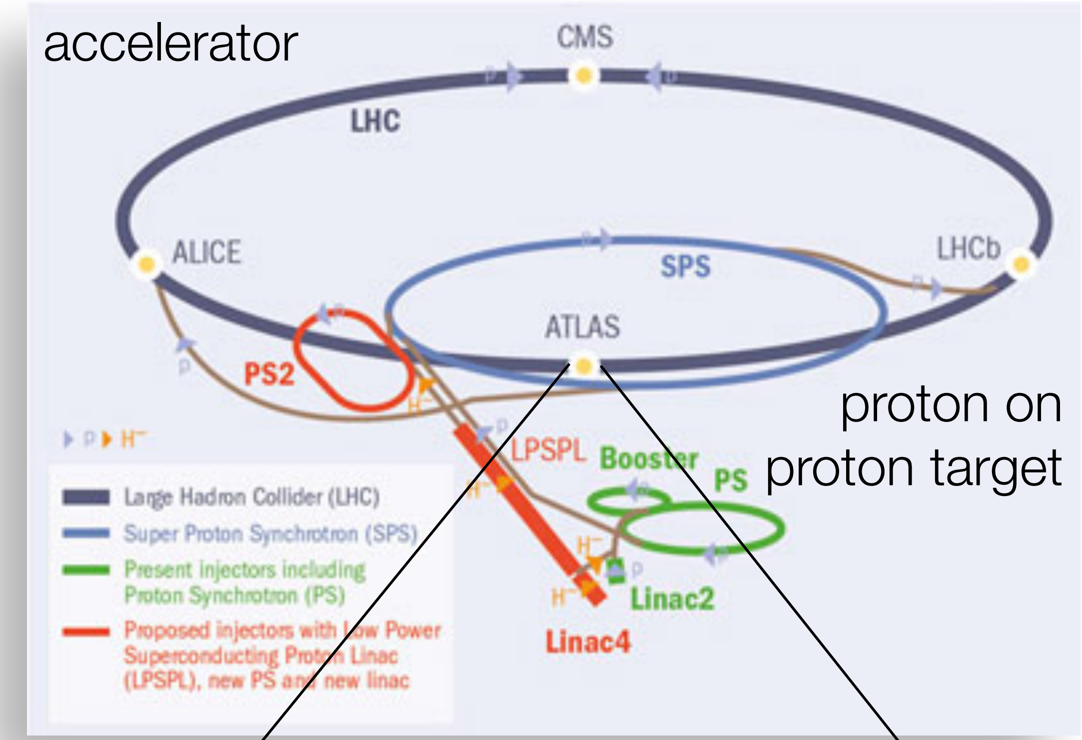
3. hadronic showers

outline

workshop in particle physics

- particle physics and cosmic rays
- electron-photon scattering
- bremsstrahlung
- electron-positron pair production
- simple derivation (virtual photon method)

particle physics
collider experiments



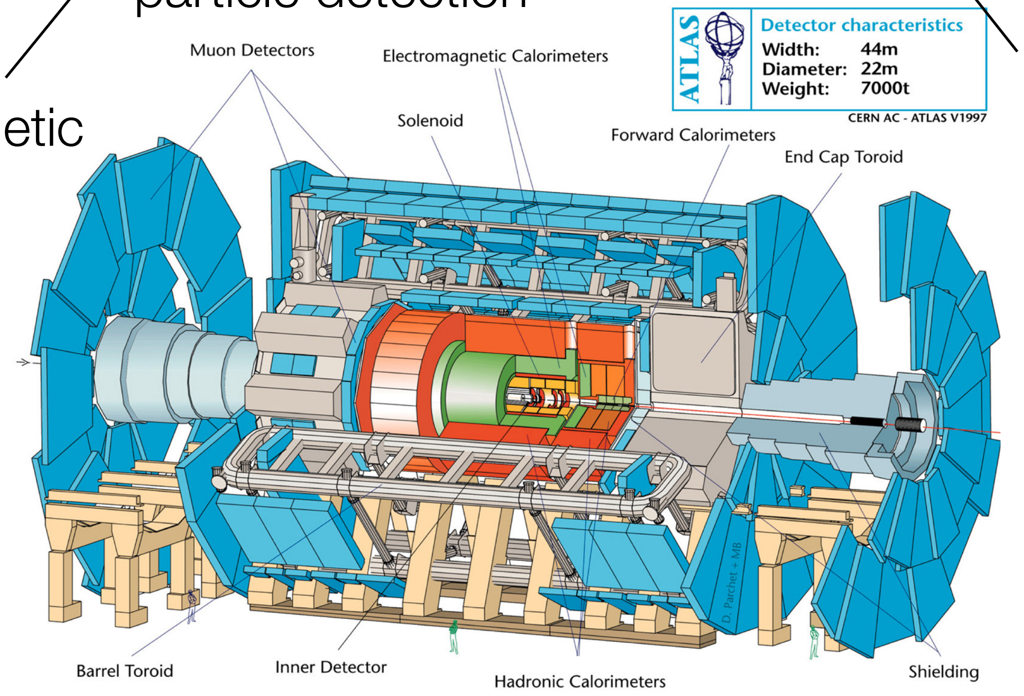
accelerator

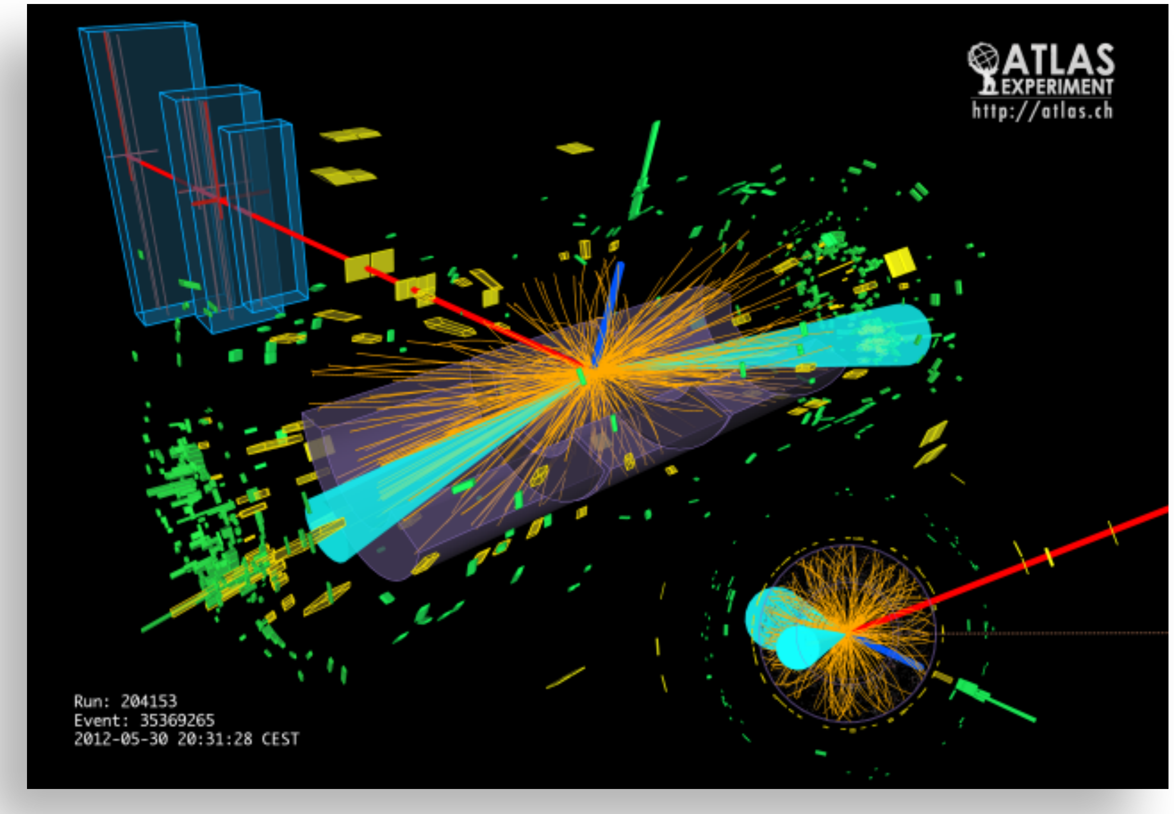
proton on
proton target

particle propagation

particle detection

magnetic
fields



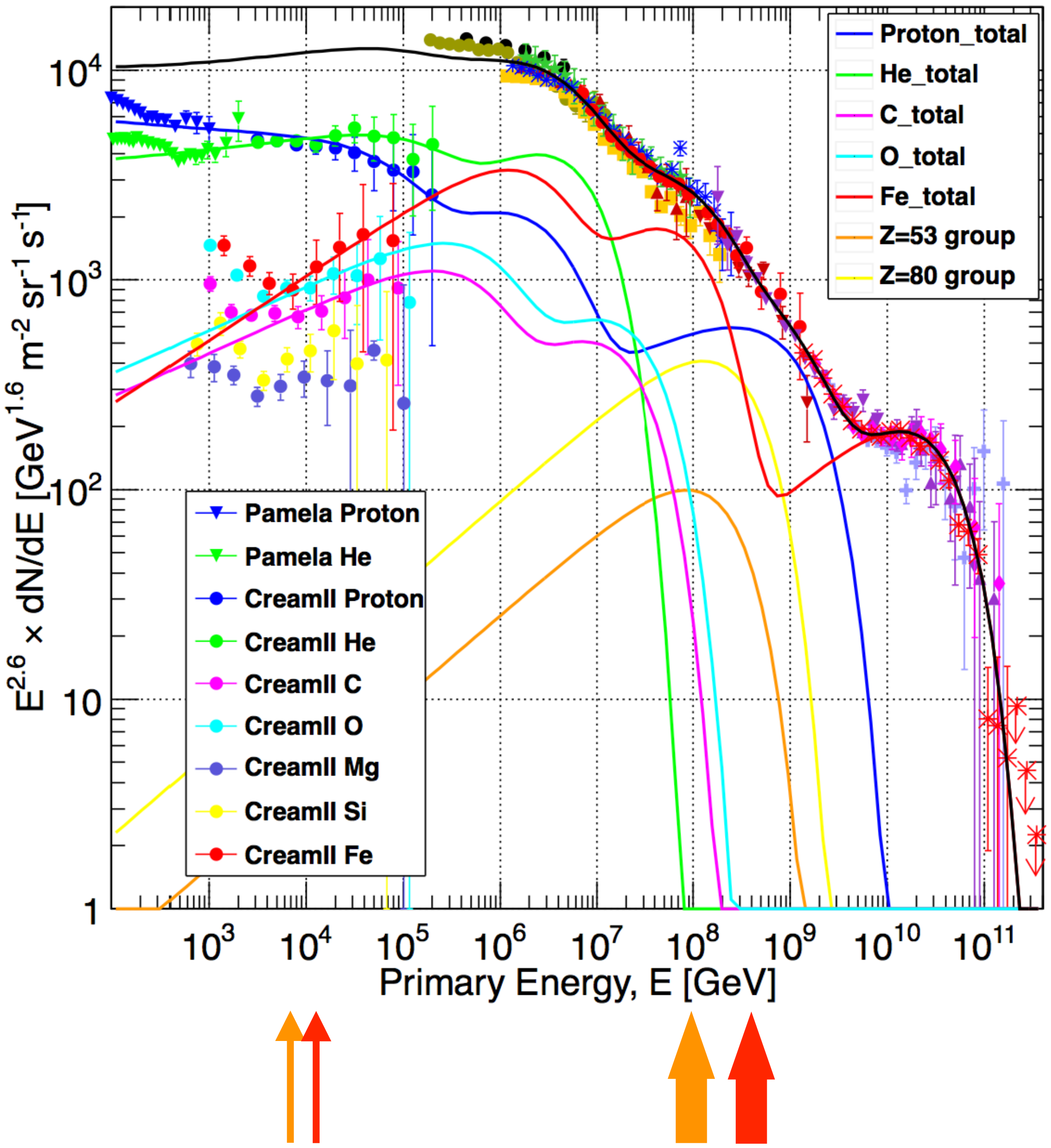


- dedicated collider laboratories
- **controlled environment**
- study particle interactions in great detail
- LHC @ c.m. energy of **7 TeV** & **14 TeV**

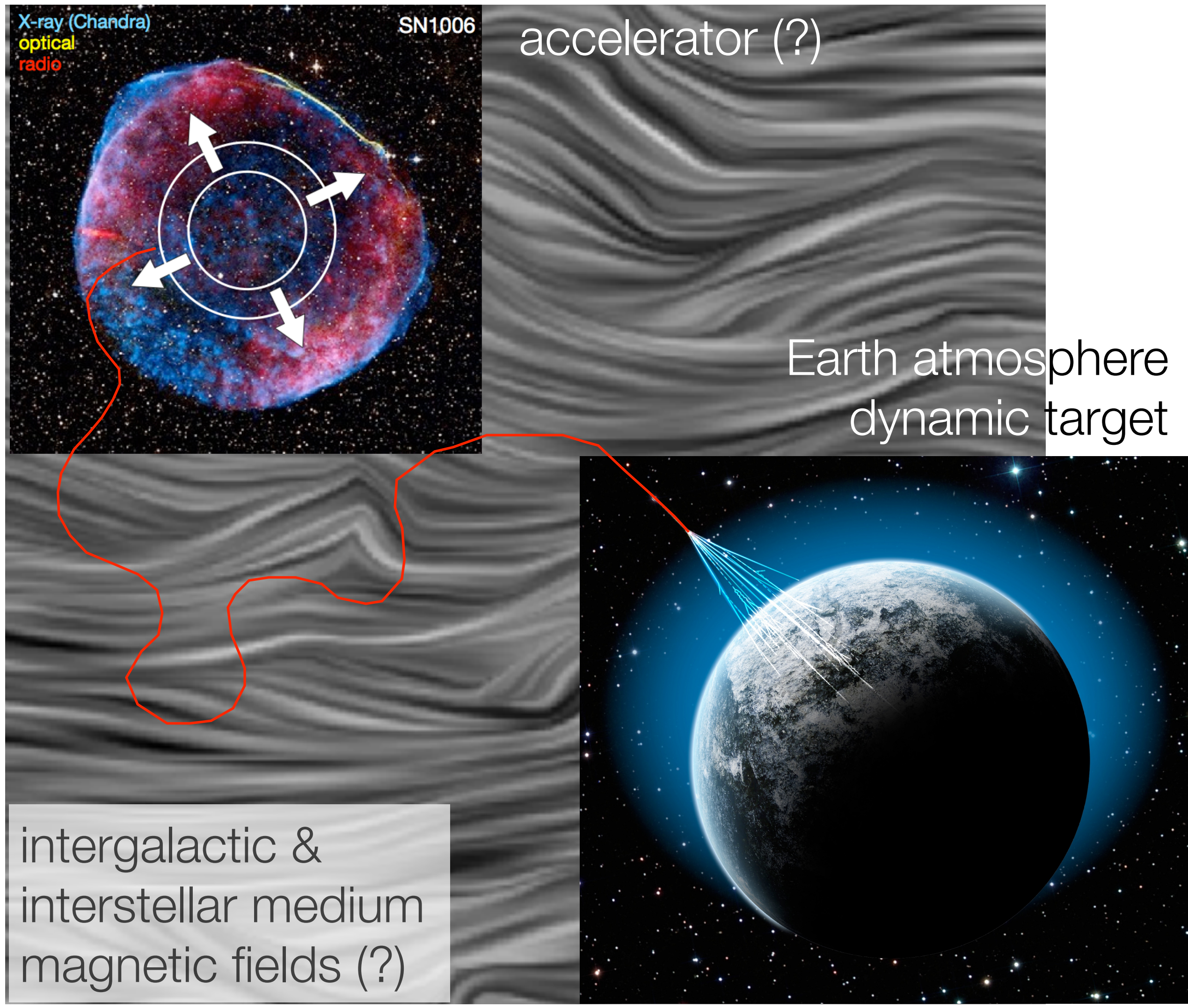
particle physics

cosmic rays

Gaisser, Stanev, Tilav: arXiv:1303.3565



LHC 7 TeV 14 TeV



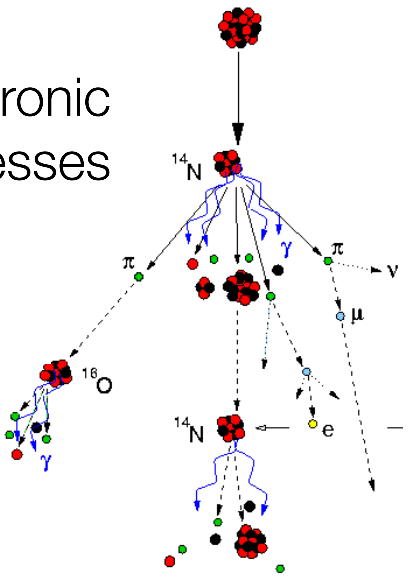
accelerator (?)

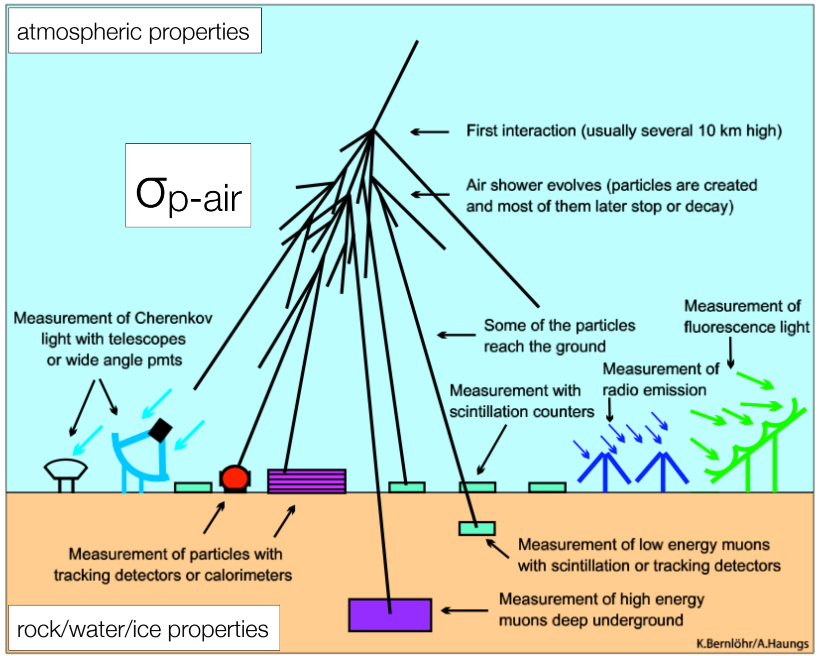
Earth atmosphere
dynamic target

intergalactic &
interstellar medium
magnetic fields (?)

propagation

nuclear & hadronic
processes

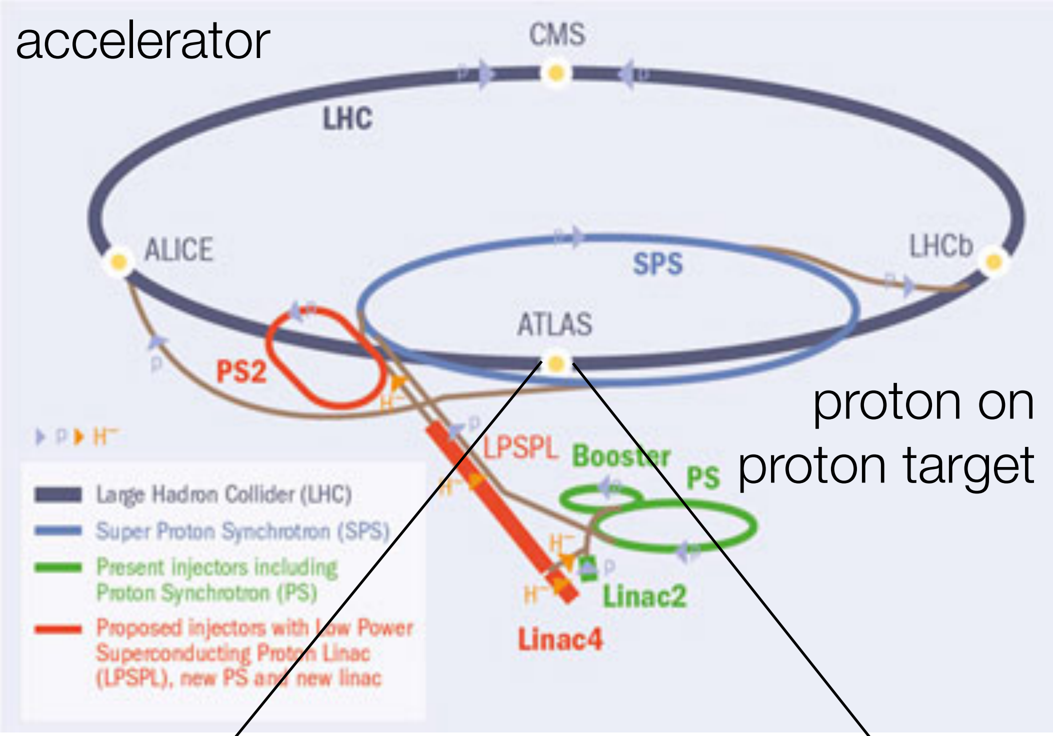




cosmic ray detection

cosmic rays
a natural laboratory

accelerator

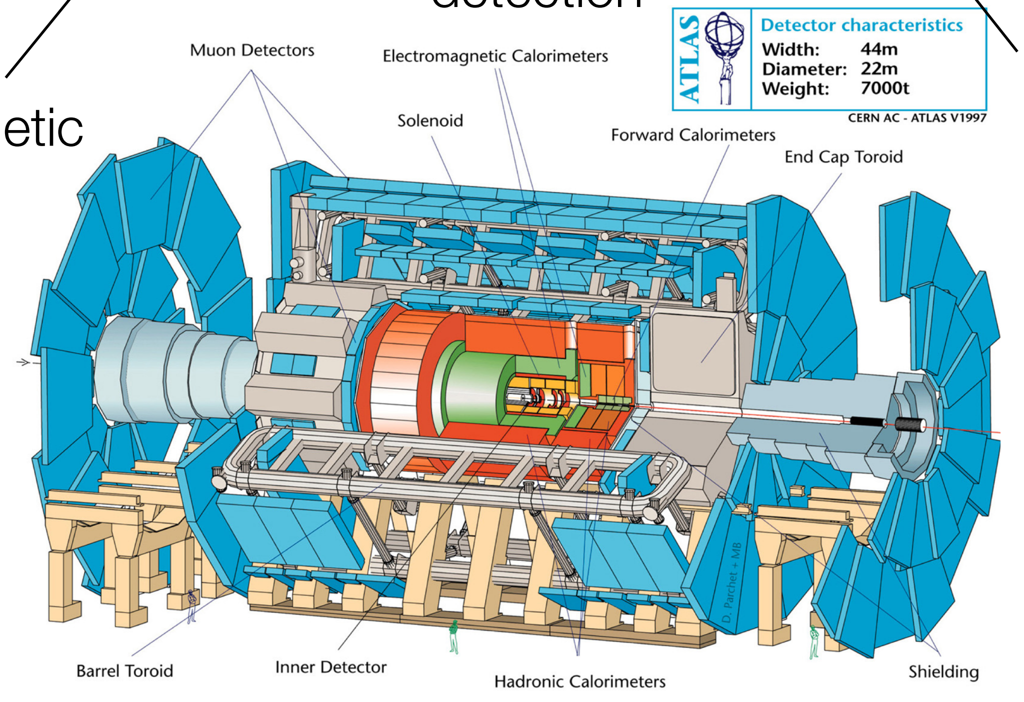


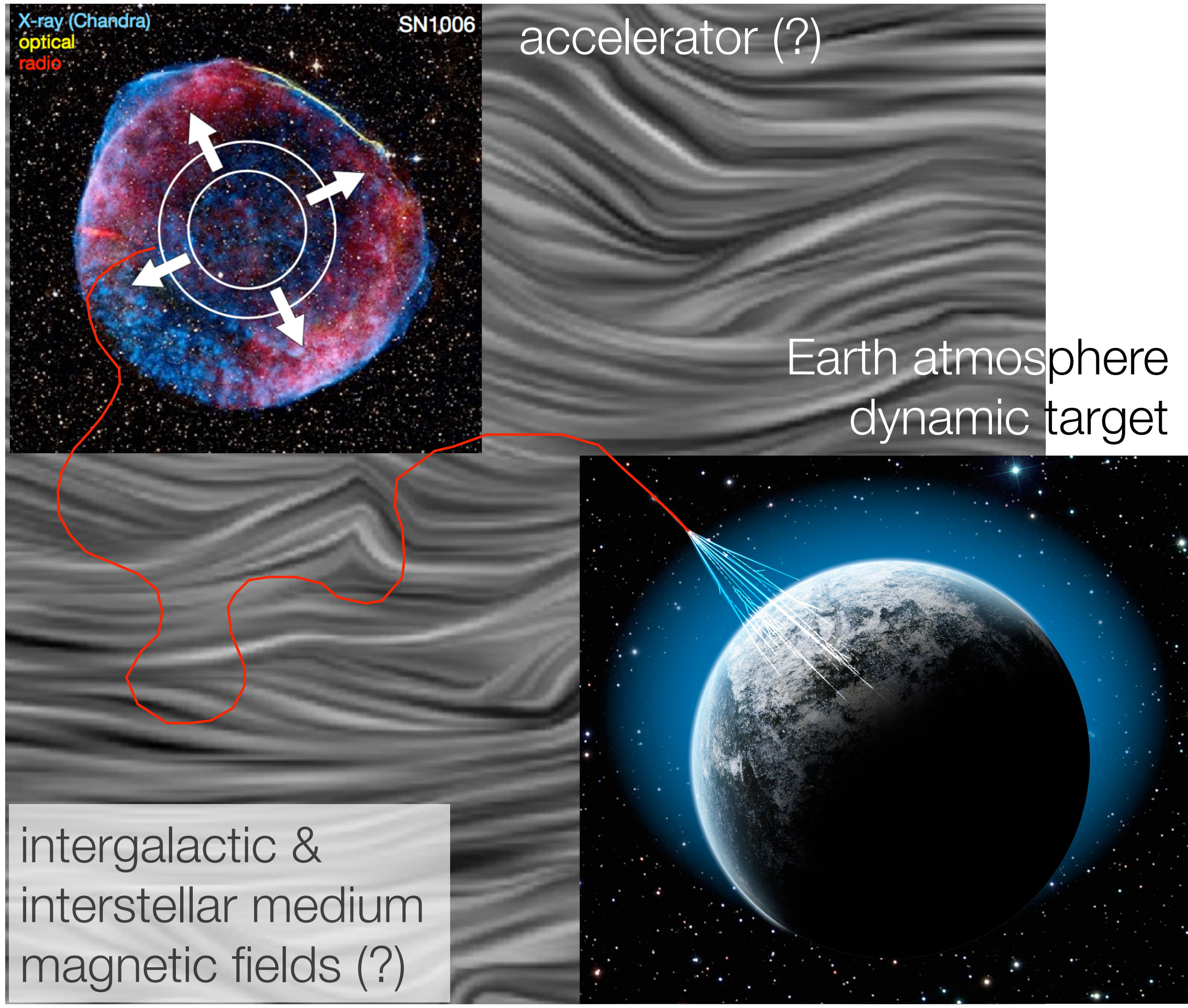
proton on
proton target

propagation

detection

magnetic
fields





accelerator (?)

Earth atmosphere
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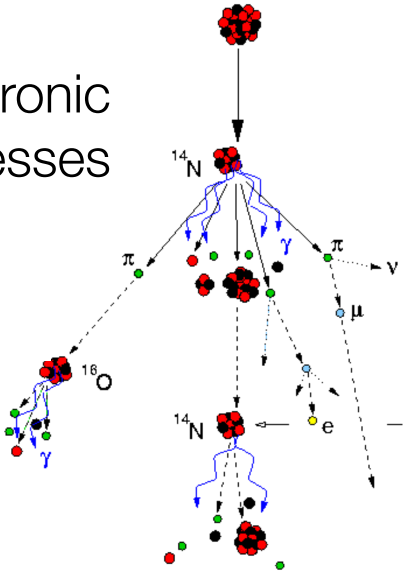
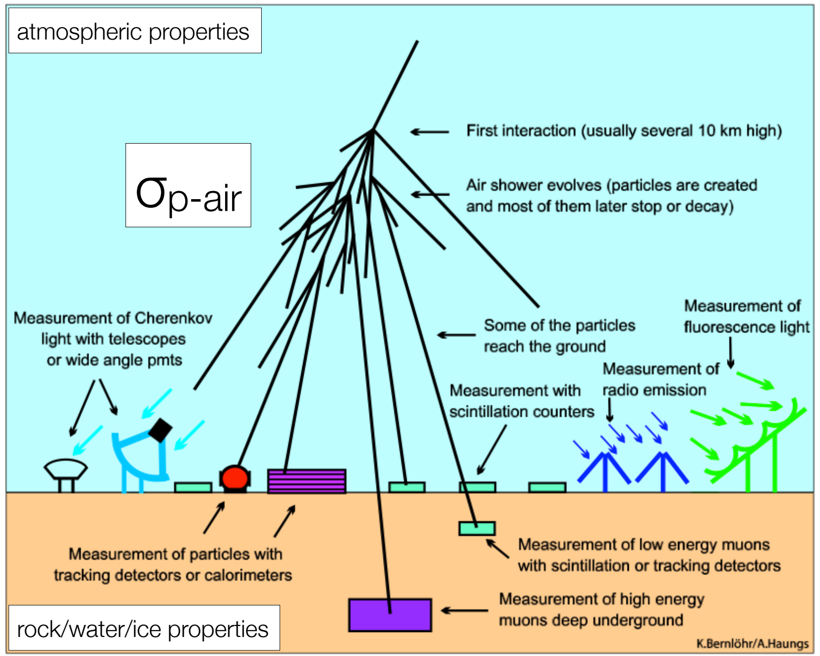




**cosmic ray
detection**

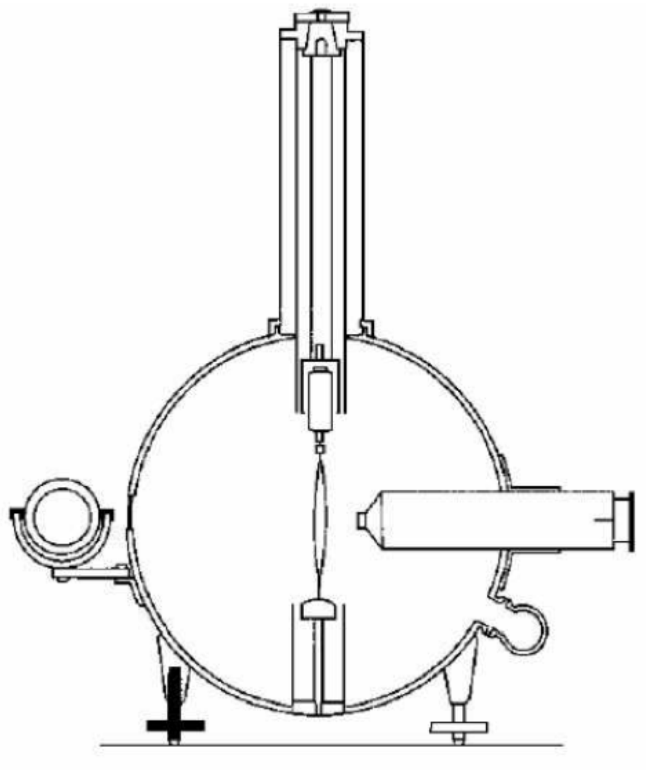
history of cosmic ray physics

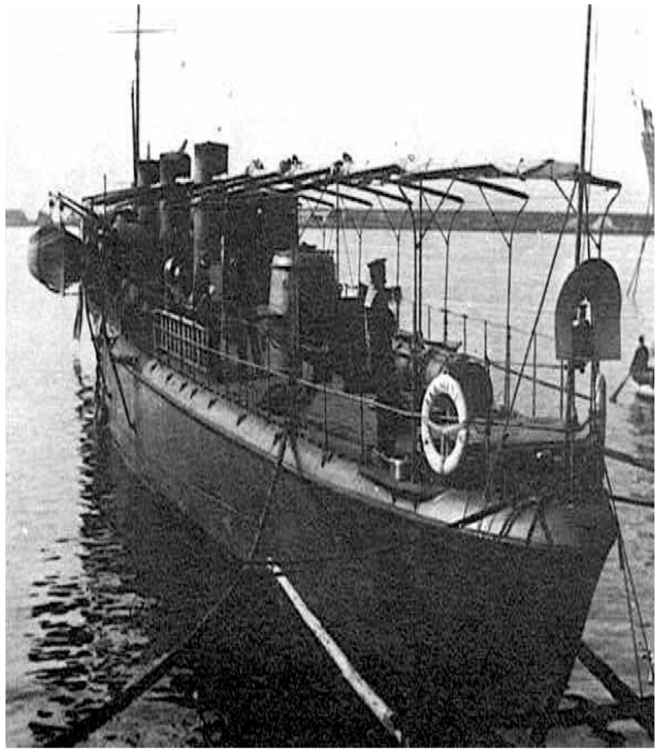
penetrating cosmic radiation

Theodor Wulf (1868-1946)

Domenico Pacini (1878-1934)

Victor Francis Hess (1883-1964)





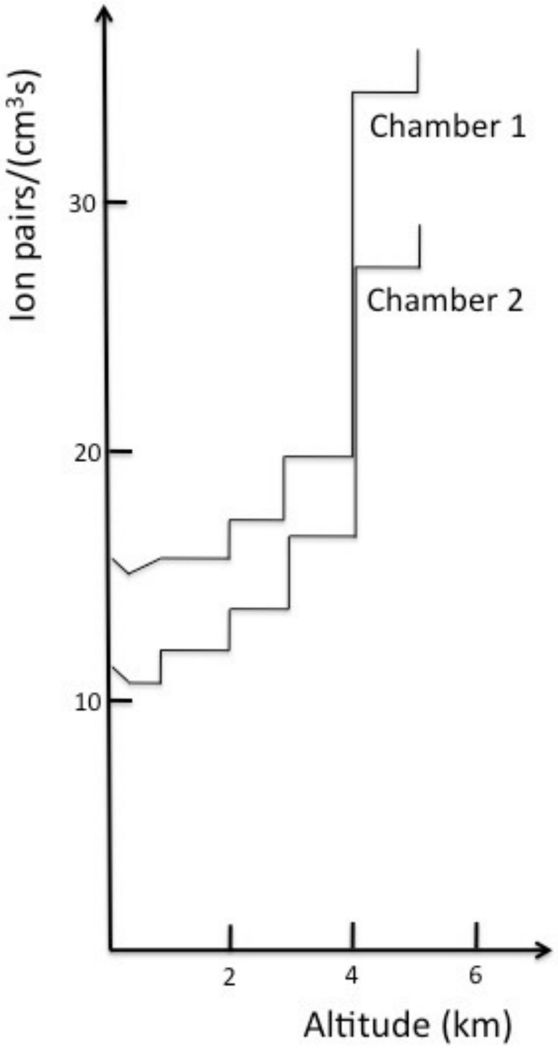
1910: radiation at the bottom and the top of Eiffel Tower. Lower decrease than expected

1910-11: radiation over and below sea surface. Decrease underwater. Seasonal variations.

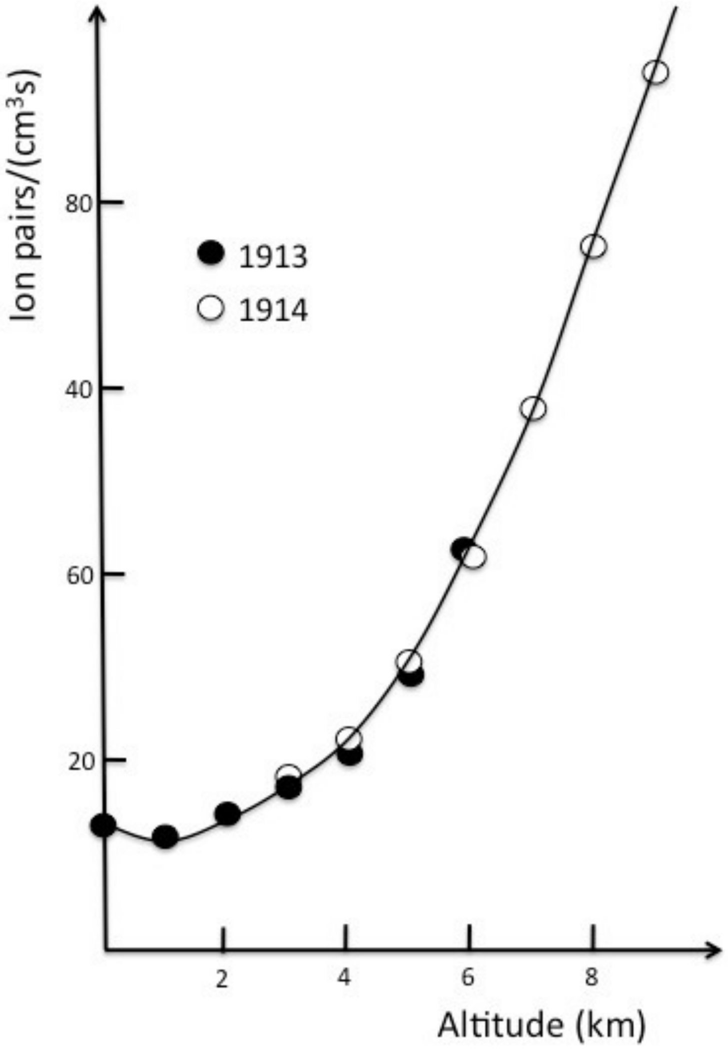
1911-12: balloon measurements up to 5,300 meters. Radiation increased 4 times.

history of cosmic ray physics

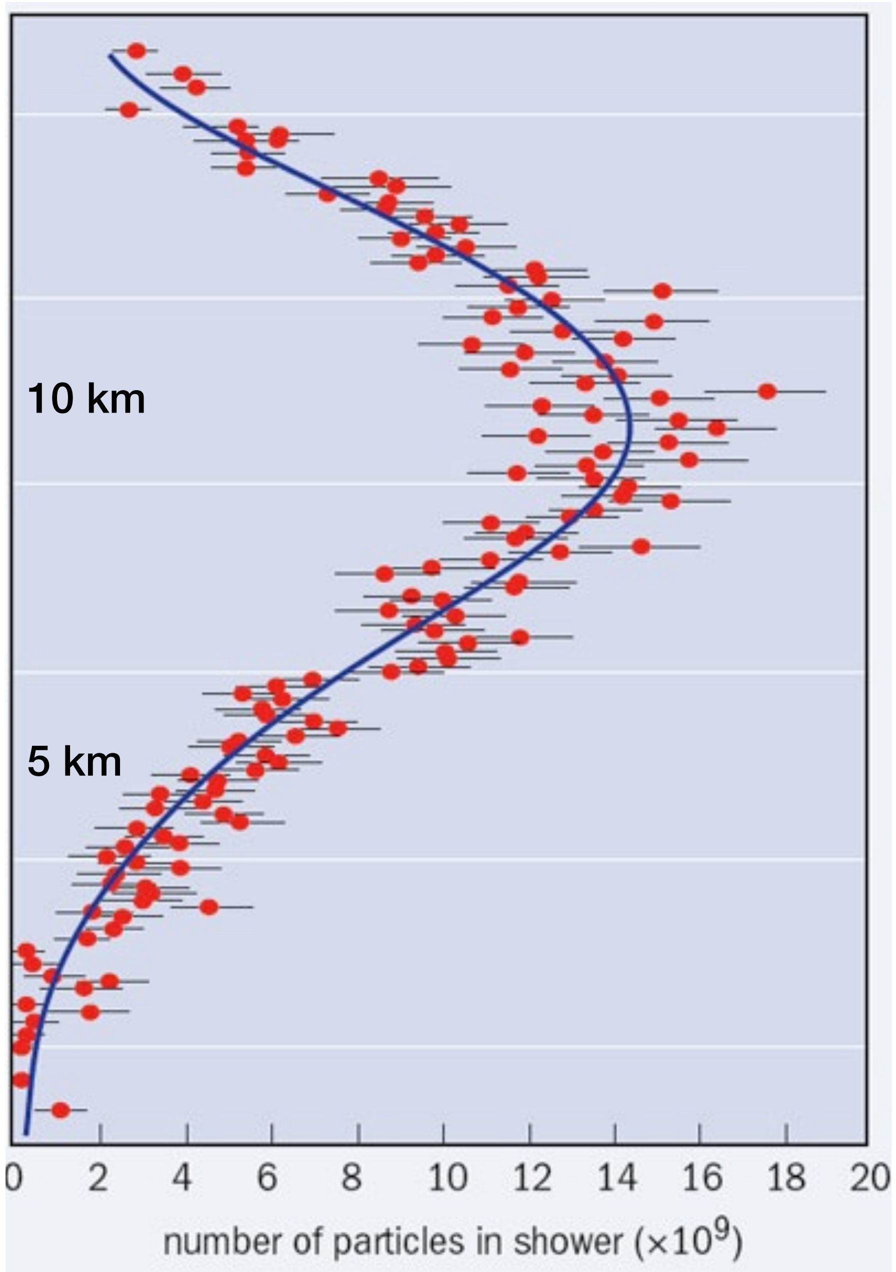
bombarding Earth from space



Hess (1912)



Kolhörster (1913-14)



history of cosmic ray physics

the nature of cosmic particles

Robert Millikan
(1868-1953)

Arthur Compton
(1892-1962)

Bruno Rossi
(1905-1993)

Pierre Auger
(1899-1993)

Louis Leprince-Ringuet
(1901-2000)

1932: strong debate between Millikan and Compton on whether cosmic rays are composed of high energy **photons** (Millikan's view) or **charged particles** (Compton's view).

Photons (gamma rays) would be produced in interstellar space by hydrogen fusion into heavier nuclei.

1930: Bruno Rossi predicts **east-west** asymmetry effect should cosmic particles be charged

1927: Clay reports latitude variations of cosmic ray flux (from Amsterdam to Indonesia).

1933: Auger and Leprince-Ringuet measure cosmic ray variations from Le Havre to Buenos Aires and find a minimum at the equator. **Cosmic particles** are predominantly **charged**.

1934: Auger and Leprince-Ringuet measure an excess of particles from West. **Cosmic particles** are predominantly **positively charged**

However, the name **cosmic rays** introduced by Millikan, will remain.

charge of cosmic rays

east-west effect

- geomagnetic field effect

- higher cosmic ray flux from west indicate particles are predominantly positive

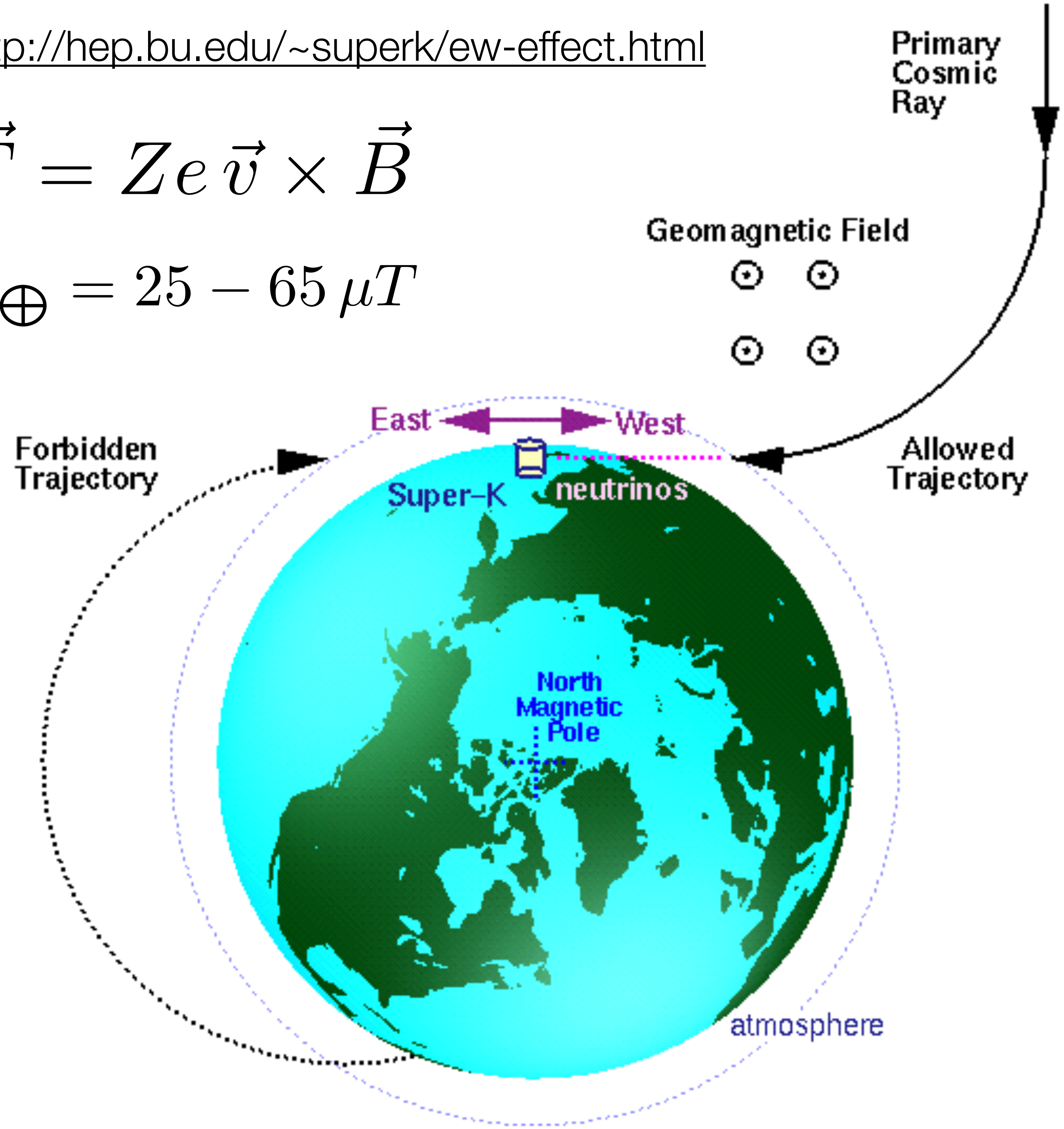
$$r_L = \frac{p_\perp}{ZeB}$$

$$\boxed{r_L \sim \frac{3.3 \times 10^4}{Z} \frac{E(GeV)}{B(G)} m}$$

http://hep.bu.edu/~superk/ew-effect.html

$$\vec{F} = Ze\,\vec{v} \times \vec{B}$$

$$B_\oplus = 25 - 65\,\mu T$$



extensive air showers

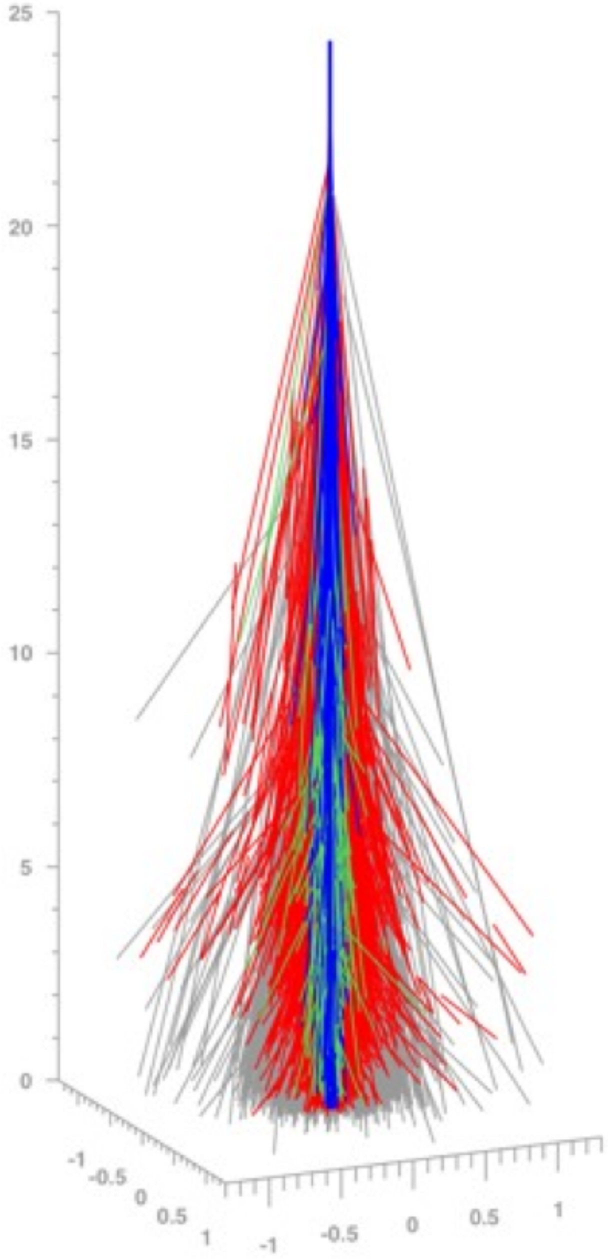
penetrating cosmic radiation

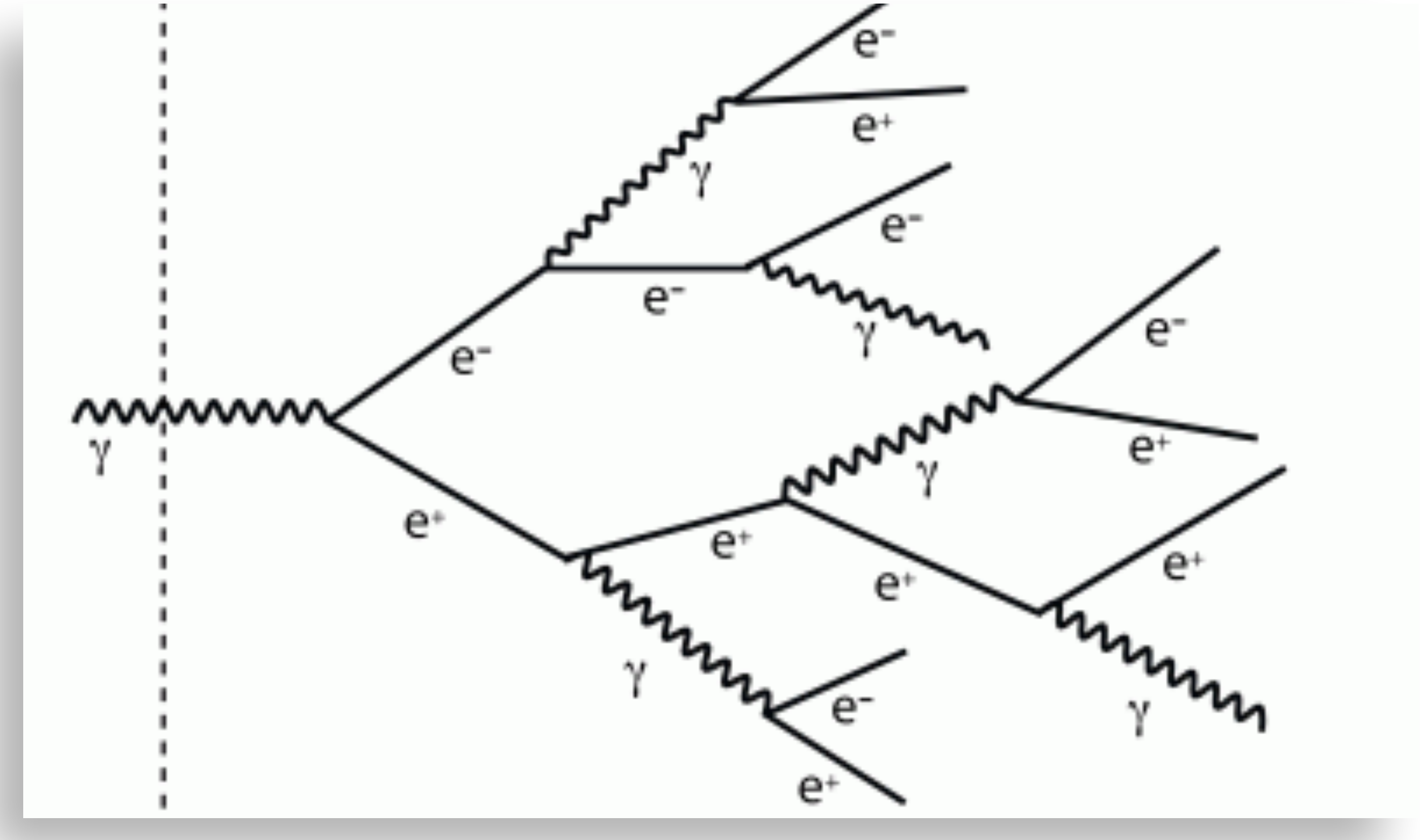
- **atmospheric air showers** of particles are **extended**
  - while measuring “east-west” effect Rossi noticed coincident far apart signals
  - independently Auger (1937) concluded that primary CR interact in upper atmosphere initiating cascade of **secondary interactions** that reaches ground

- Bhabha & Heitler (1937) explained development of **soft** CR showers as sequence of **γ rays** and **e^-e^+ pairs**

- evidence of **hard** penetrating component of **hadronic** CR showers that can be detected underground

proton-induced shower of 10^{19} eV

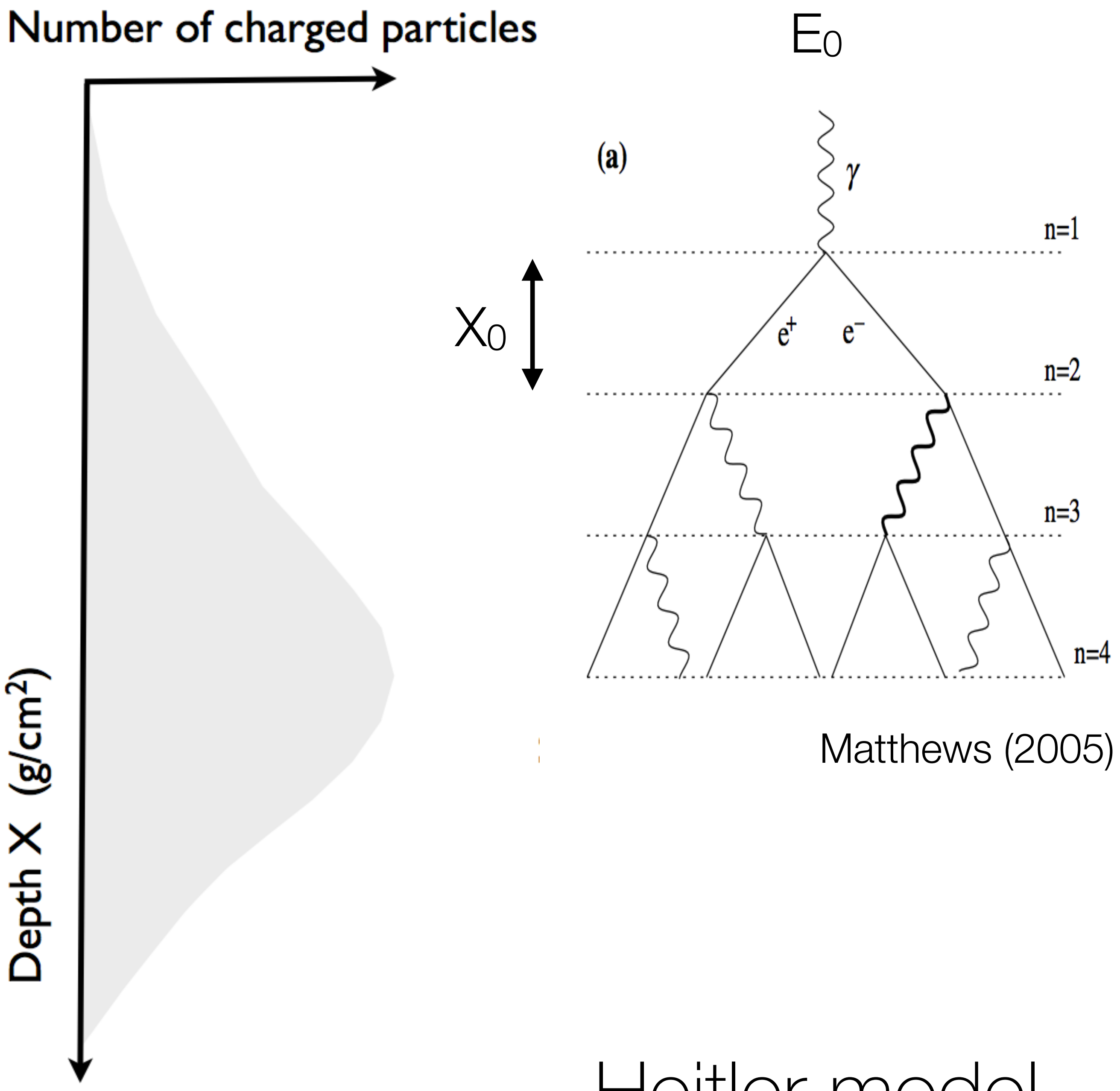




ELECTROMAGNETIC SHOWERS

electromagnetic showers

- particles involved
  - ***e*± *and* γ**
- interactions involved
  - *bremsstrahlung*
  - *pair production*
  - *ionization losses*



Matthews (2005)

Heitler model

fundamental processes
in quantum electrodynamics

$$e^{\pm} + \gamma \longrightarrow e^{\pm} + \gamma$$

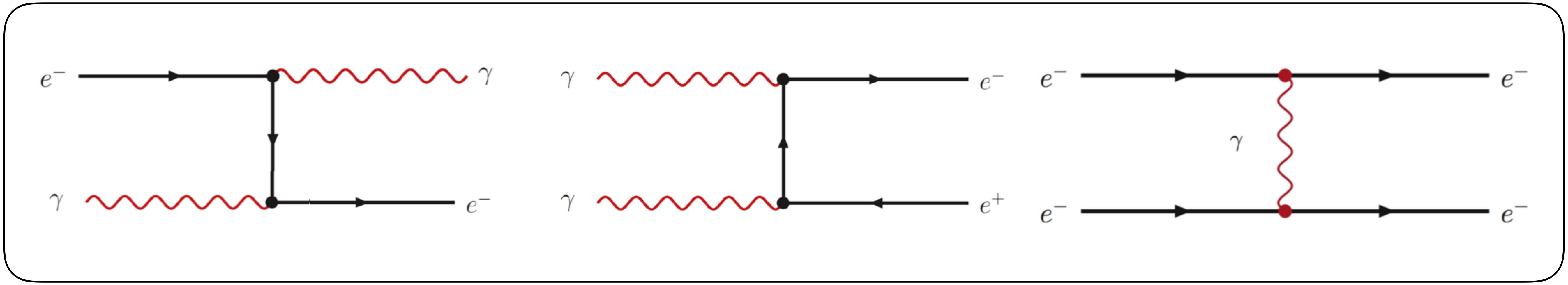
COMPTON SCATTERING

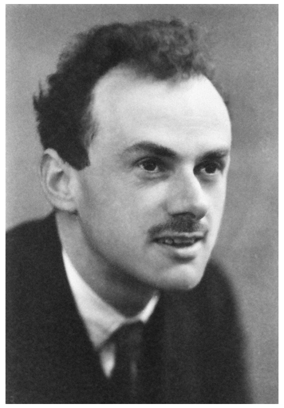
$$\gamma + \gamma \rightarrow e^{+} + e^{-}$$

ELECTRON-POSITRON CREATION (ANNIHILATION)

$$e + e \rightarrow e + e$$

ELECTRON-POSITRON SCATTERING

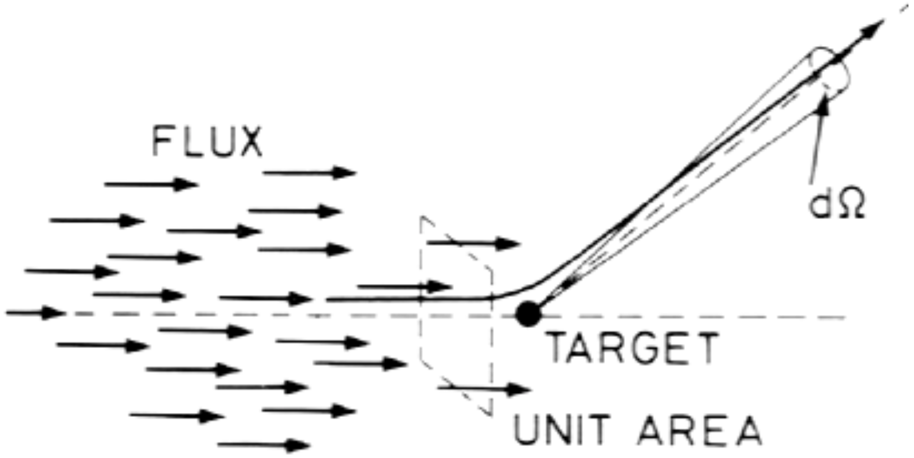




1930: Paul Dirac postulates the existence of **positron**

Cross Section

and differential cross section



- effective area providing the probability of some scattering event or the likelihood of interaction between particles

flux of incident particles $$\Phi = \frac{dN_{\text{beam}}}{dA\,dt}[\text{m}^{-2}\text{sec}^{-1}]$$

- number of **interactions/sec**

$$\frac{dN_{\text{int}}}{dt} = \Phi\, n_{\text{target}}\, \sigma$$

- number of *scattering centers*

$$n_{\text{target}} = \rho_{\text{target}}\, \frac{N_A}{A}\, \text{Area}\, \delta$$

- the number of **interactions/sec/sr**

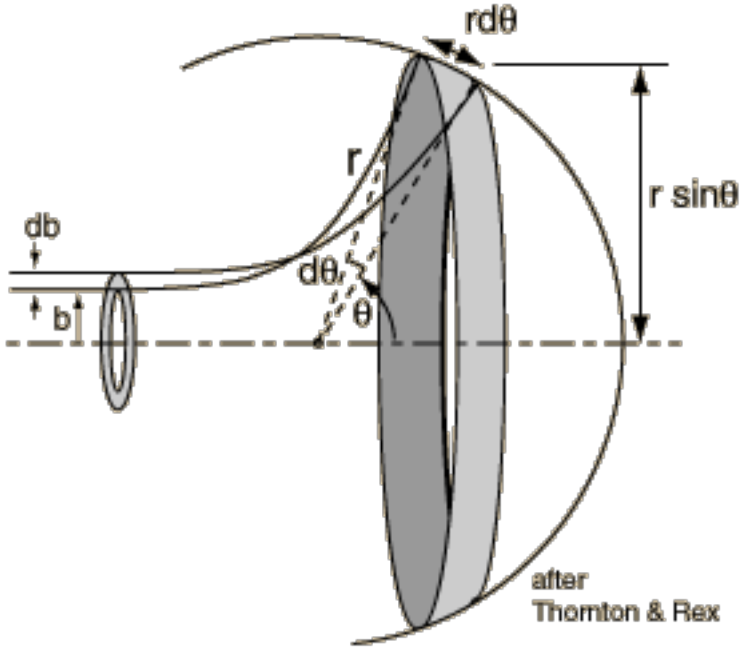
$$\frac{dn}{d\Omega} = \Phi\, n_{\text{target}}\, \frac{d\sigma}{d\Omega}$$

Cross Section

and differential cross section

- on a sphere

$$\frac{d\sigma}{d\Omega} = \frac{b}{sin\theta}\left|\frac{db}{d\theta}\right|$$



after Thornton & Rex

- … Rutherford scattering experiment (discovery of the atomic nucleus)

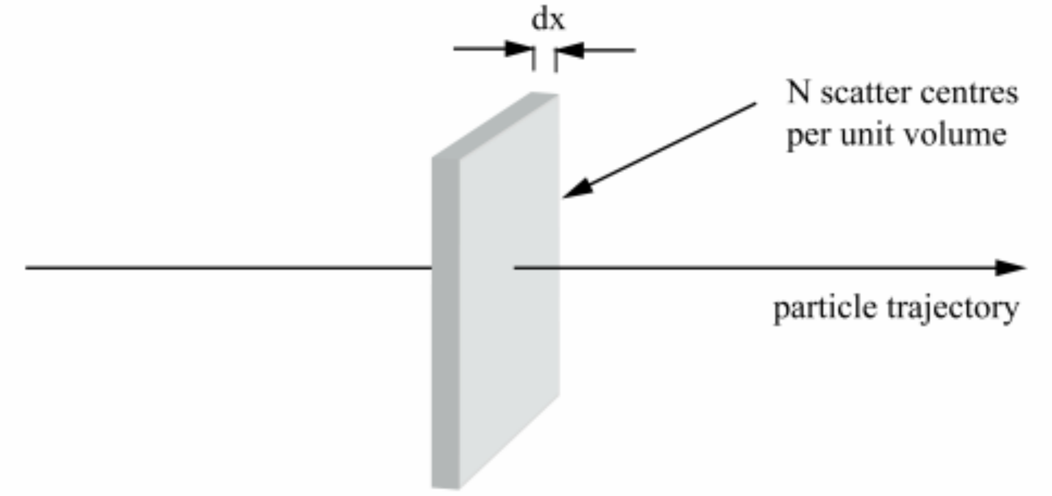
- $b = \left(\frac{Z_1\, Z_2\, e^2}{4\pi\, \epsilon_0\, E}\right)\cot\frac{\theta}{2}$

$$\boxed{\frac{d\sigma}{d\Omega} = \left(\frac{Z_1\, Z_2\, e^2}{16\pi\, \epsilon_0\, E}\right)^2 \frac{1}{\sin^4\frac{\theta}{2}}}$$

Rutherford cross section

Cross Section

mean free path



- interaction probability

$$W(X) = \hat{n}\,\sigma\,e^{-\hat{n}\,\sigma\,x}$$

$$dW = \hat{n}\,\sigma\,dx$$

$$\hat{n} = \rho_{\text{target}}\frac{N_A}{A} = \frac{\rho_{\text{target}}}{\langle m_{\text{target}}\rangle}$$

number density of scattering centers

cross section

target thickness

- mean free path (interaction length)

$$\lambda[m] = \int_0^\infty W(E)\,x\,dx = \frac{1}{\hat{n}\sigma}$$

$$\lambda_{\text{int}}[g/cm^2] = \frac{\rho_{\text{target}}}{\hat{n}\,\sigma} = \frac{\langle m_{\text{target}}\rangle}{\sigma}$$

Lorentz covariance

and Mandelstam variables

- Lorentz covariance is a property of **spacetime** following from **special relativity**. Physics quantities do not change with reference system

- a physical quantity is Lorentz covariant if it transforms under Lorentz transformations

  - in spacetime displacement is represented by 4-vector $X^\mu = (ct, x, y, z)$

  - velocity by 4-vector $$U^\mu = \frac{dX^\mu}{d\tau} = \gamma \left(c, \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \right)$$

  - momentum by 4-vector $$P^\mu = m_0 U^\mu = \left(\frac{E}{c}, p_x, p_y, p_z \right)$$

  - 4-momentum is conserved

Lorentz covariance

and Mandelstam variables

- Lorentz transformation preserve space-time interval

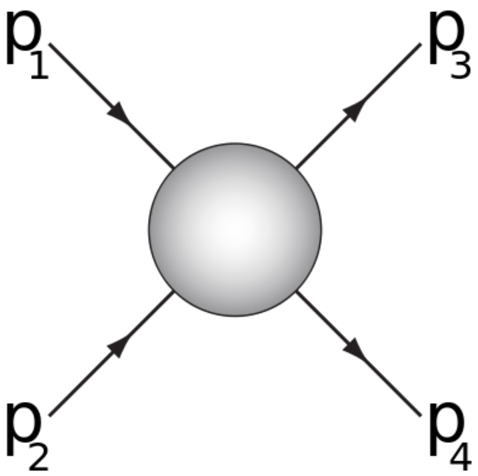
$$ds^2 = X^\mu X^\nu \eta_{\mu\nu} = c^2 dt^2 - dx^2 - dy^2 - dz^2$$

- proper time

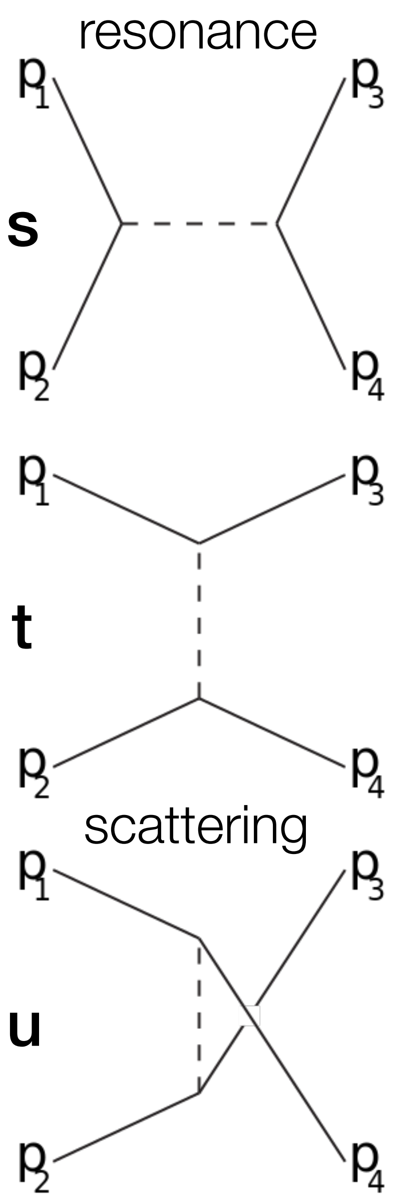
$$d\tau^2 = \frac{dt}{\gamma} = \frac{ds^2}{c^2}$$

- rest mass (invariant mass) $m_0^2 c^2 = P^\mu P^\nu \eta_{\mu\nu} = \frac{E^2}{c^2} - p_x^2 - p_y^2 - p_z^2$

Mandelstam variables



- numerical quantities encoding particles energy, momentum and angles in scattering processes in a Lorentz invariant formalism



- s-process

$$s = (P_1^\mu + P_2^\mu)^2 = (P_3^\mu + P_4^\mu)^2$$

invariant mass

- t-process

$$t = (P_1^\mu - P_3^\mu)^2 = (P_2^\mu - P_4^\mu)^2$$

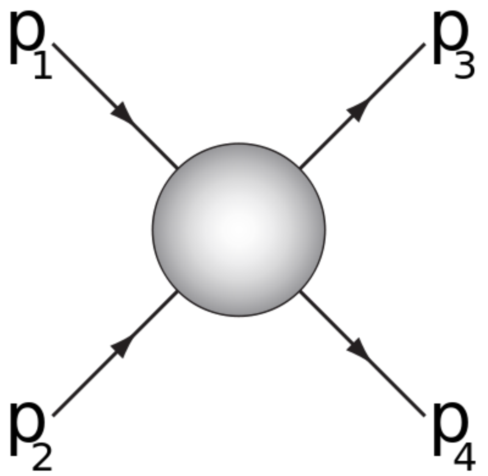
transferred 4-momentum

- u-process

$$u = (P_1^\mu - P_4^\mu)^2 = (P_2^\mu - P_3^\mu)^2$$

transferred 4-momentum

Mandelstam variables

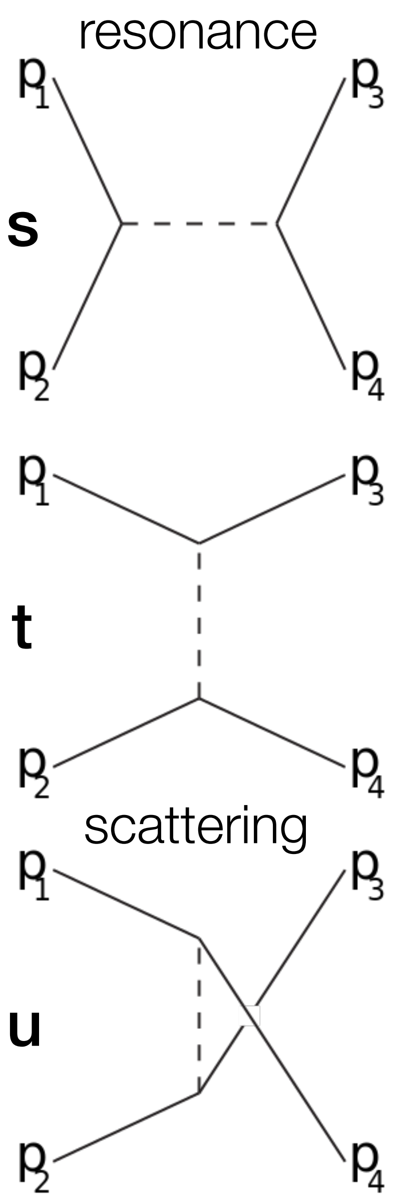


- numerical quantities encoding particles energy, momentum and angles in scattering processes in a Lorentz invariant formalism

- energy @ center of mass $E_1 = \frac{1}{2\sqrt{s}}[s + (m_1c^2)^2 - (m_2c^2)^2]$

- scattering angle @ center of mass $\cos\theta = 1 + \frac{t}{|\vec{p}|^2}$

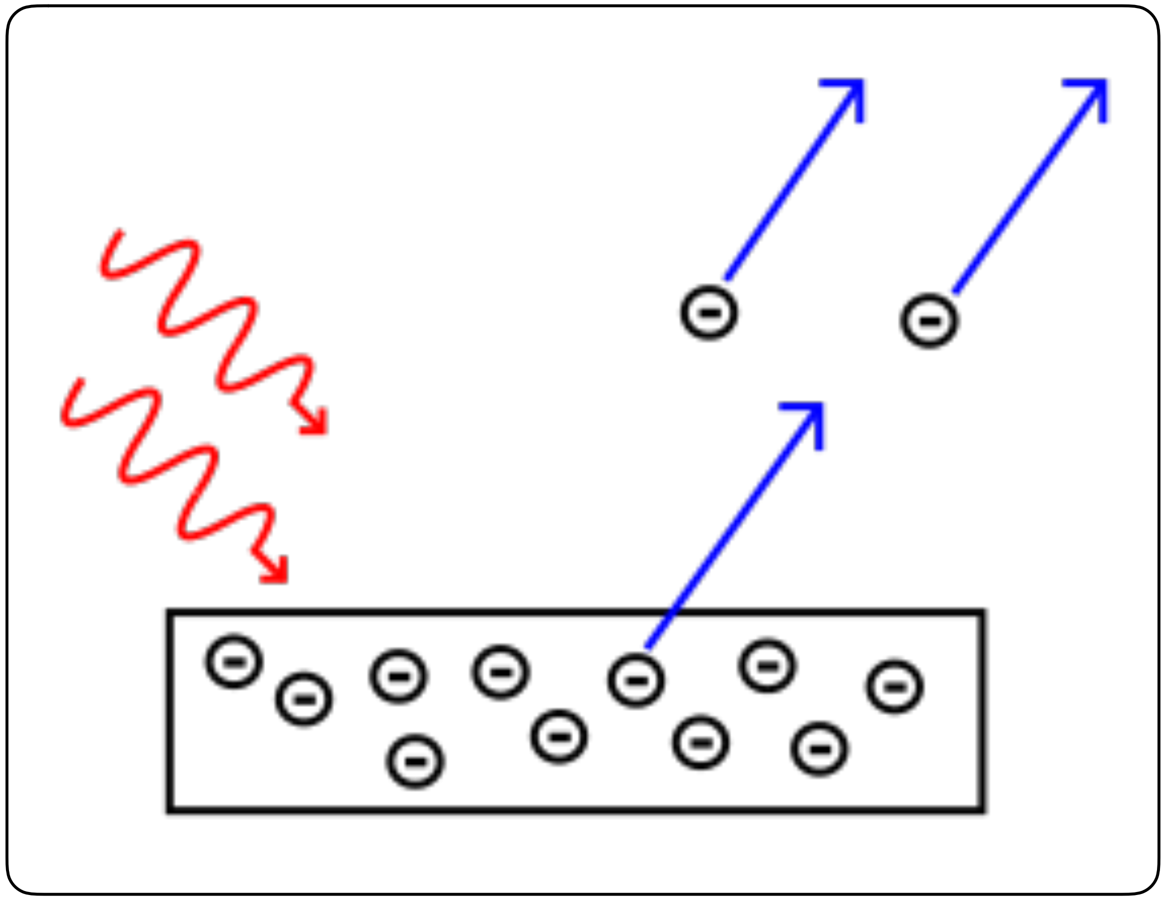
- center of mass → lab $E_{lab} = \frac{2E_{cm}^2 - (mc^2)^2}{mc^2}$



γe → γe scattering

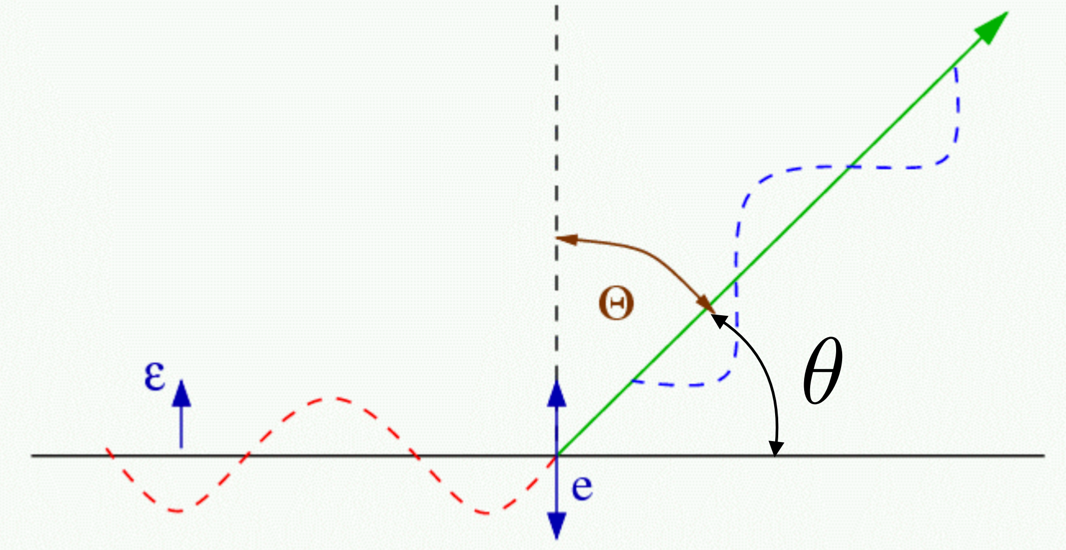
photoelectric effect

- **E_γ** « **$m_e c^2$** on electrons **bound** in atoms
  - photons **eject** electrons from atoms and is **absorbed**
  - photoelectric effect (Einstein 1905): $E_\gamma = E_e^{binding} + E_e^{kinetic}$
  - photons behave as **particles**



γe → γe scattering

Thomson scattering



- **E$_\gamma$ ≪ m$_e$c^2** on **loosely bound** or free electrons
  - Thomson (elastic) scattering
  - electron accelerated by E plane wave radiating energy (classical/quantistic solution)

$$\frac{d\sigma}{d\Omega} = r_e^2 \; sin^2\Theta$$

polarized wave

$$\frac{d\sigma}{d\Omega} = r_e^2 \; \left(\frac{1+\cos^2\theta}{2}\right) = \alpha^2\, \hat{r}_c^2 \; \left(\frac{1+\cos^2\theta}{2}\right)$$

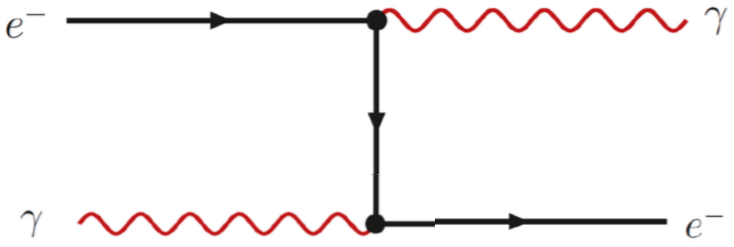
randomly polarized wave

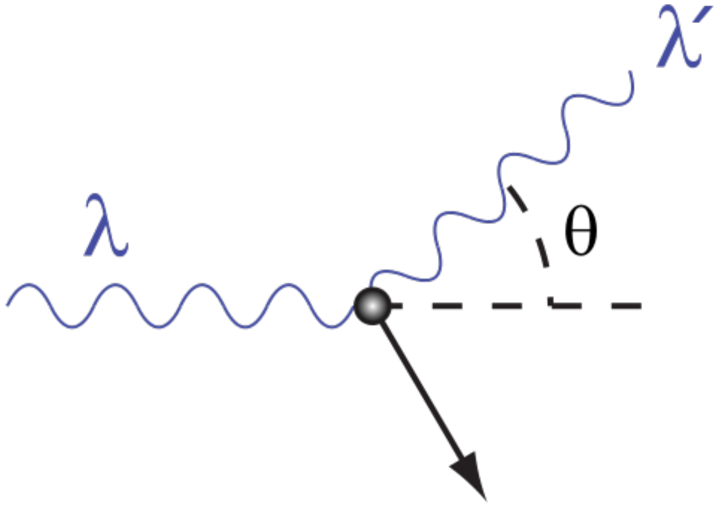
$$\sigma_T = \frac{8\pi}{3} r_e^2 \simeq 6.65 \times 10^{-25}\,\mathrm{cm}^2$$

$r_e = \frac{e^2}{mc^2}$ $\left(r_e = \frac{1}{4\pi\epsilon_0}\frac{e^2}{mc^2}\right)$ classic electron radius

$\hat{r}_c = \frac{\hbar}{mc}$ Compton electron wavelength

γe → γe scattering
Compton scattering





- **E$_\gamma$ ≳ m$_e$c^2** on free electrons
- inelastic scattering: photon transfer energy to electron
  - Compton scattering (quantistic)

$$\lambda' - \lambda = \frac{h}{m_e\,c}(1-\cos\theta)$$

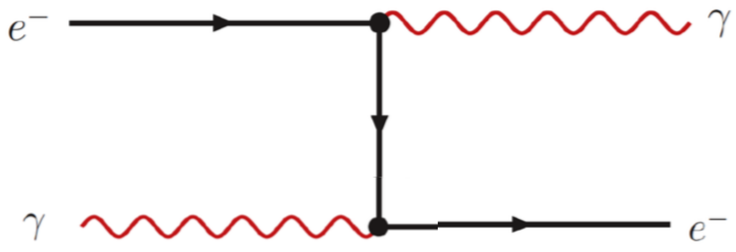
$$r_c = \frac{h}{m_e\,c}$$

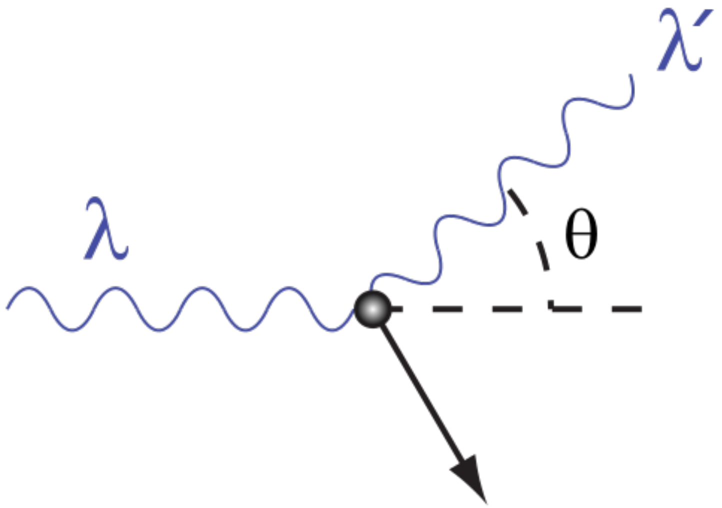
- photons *always* loose energy

1923: Arthur Compton measured the **change in wavelength** of scattered light off electrons

γe → γe scattering

Compton scattering





- **$E_\gamma \gtrsim m_ec^2$** on free electrons
- inelastic scattering: photon transfer energy to electron

▸ Klein-Nishina Formula

$$\frac{d\sigma}{d\Omega} = \alpha^2\,\hat{r}_c^2\,P(E_\gamma,\theta)^2\,\frac{1}{2}\left[P(E_\gamma,\theta) + \frac{1}{P(E_\gamma,\theta)} - 1 + \cos^2\theta\right]$$

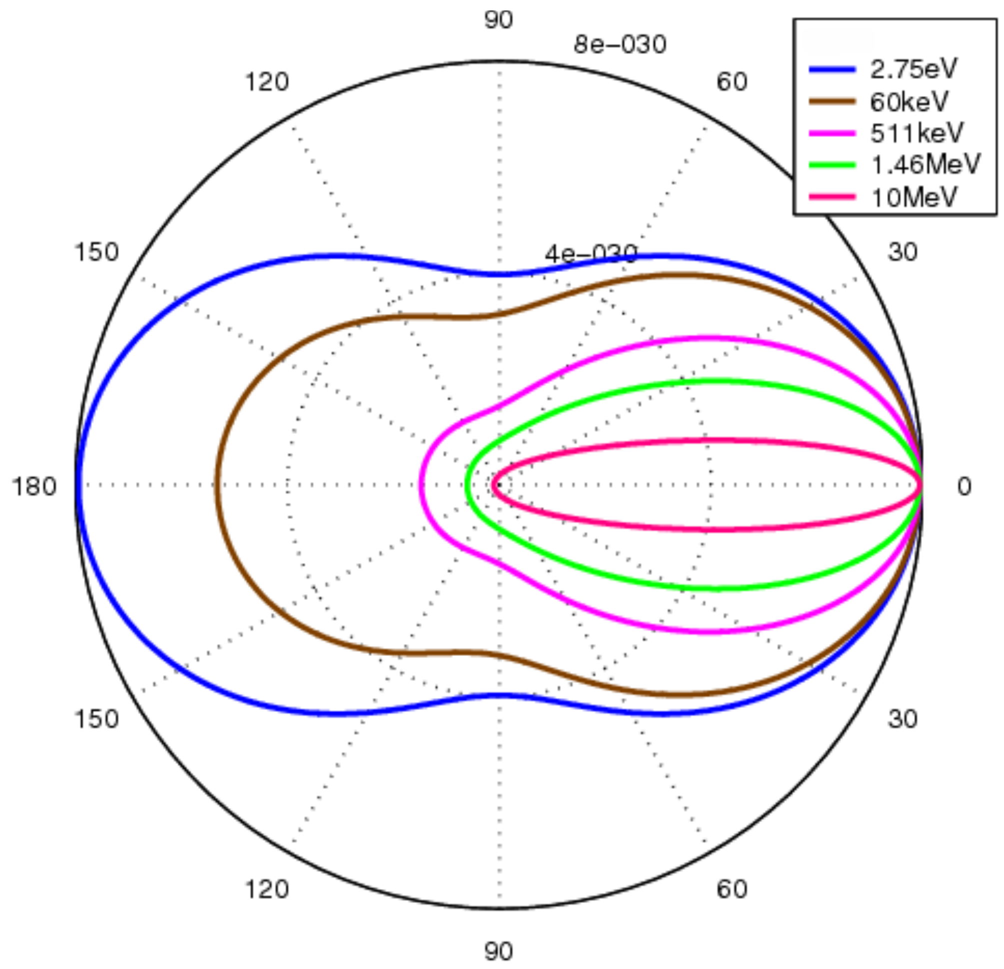
$$P(E_\gamma,\theta) = E_\gamma^{final}/E_\gamma^{initial}$$

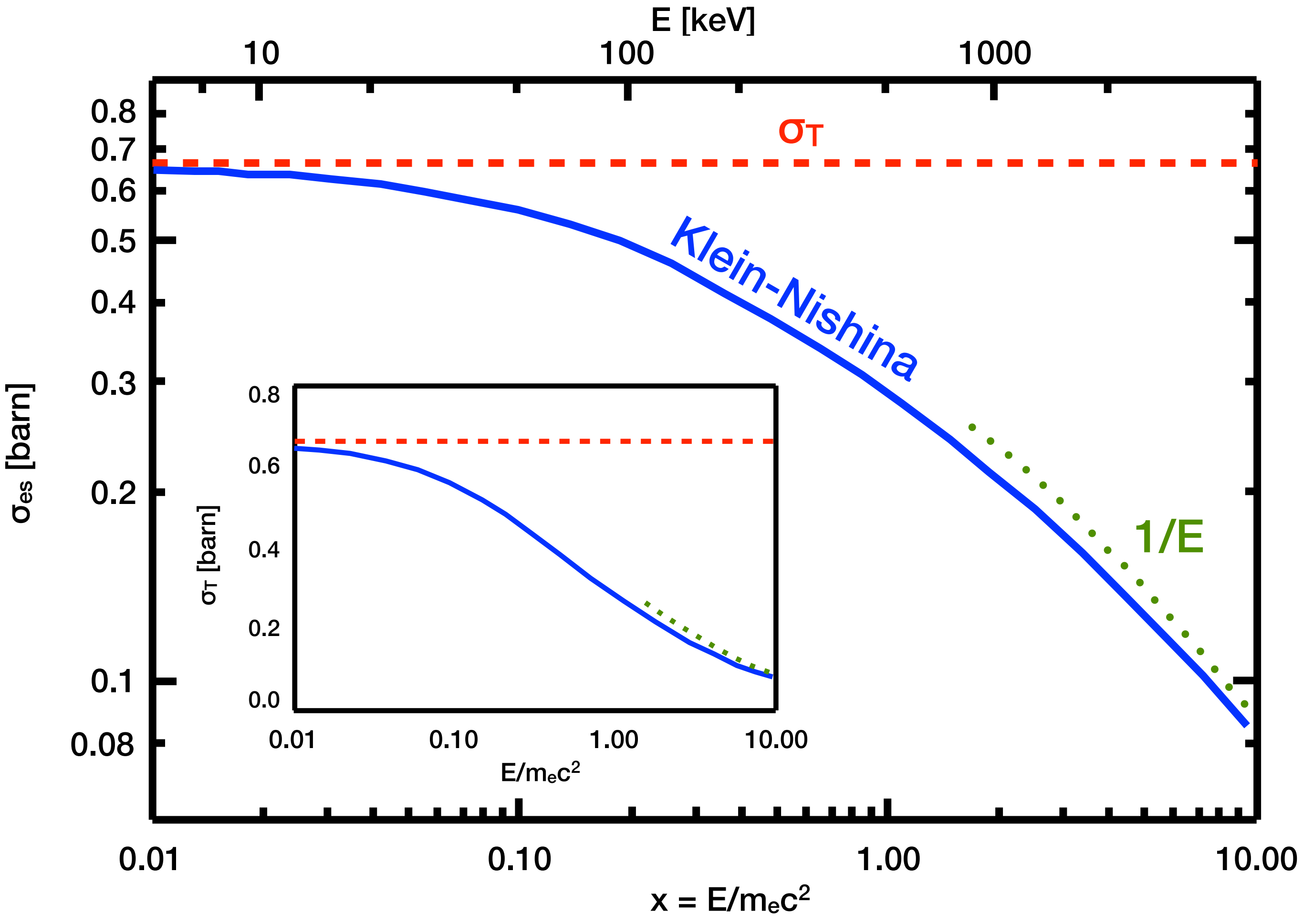
$$P(E_\gamma,\theta) = \frac{1}{1 + \frac{E_\gamma}{m_ec^2}(1-\cos\theta)}$$

γe → γe scattering

Klein-Nishina formula

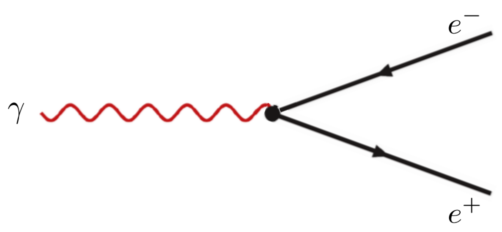
- **visible light** (Thomson scattering)
  - elastic scattering
  - dipolar angular distribution
- **X-rays, γ-rays** (Compton scattering)
  - inelastic scattering
  - forward angular distribution
  - QED reduction effects





e-pair production

electromagnetic shower

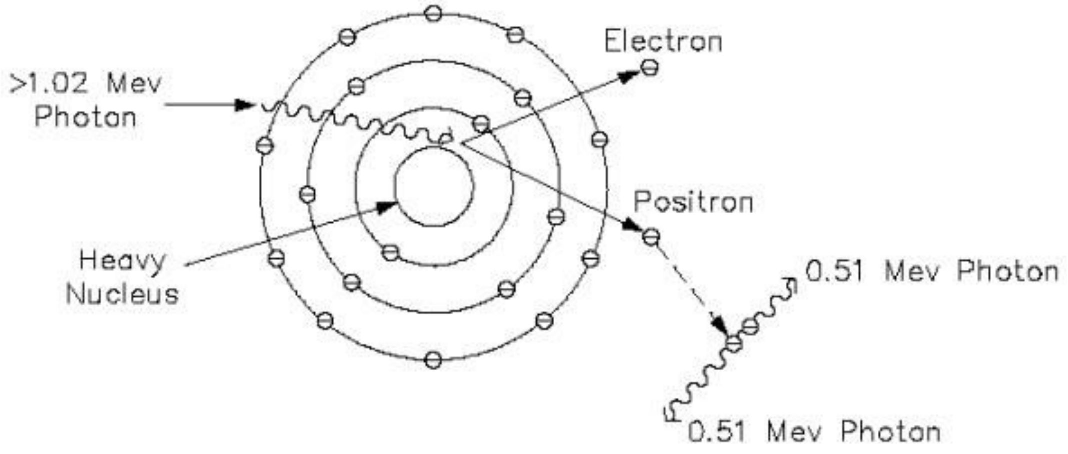


- **$E_\gamma \gg 2m_ec^2$**
- electron-positron pair production
  - threshold process

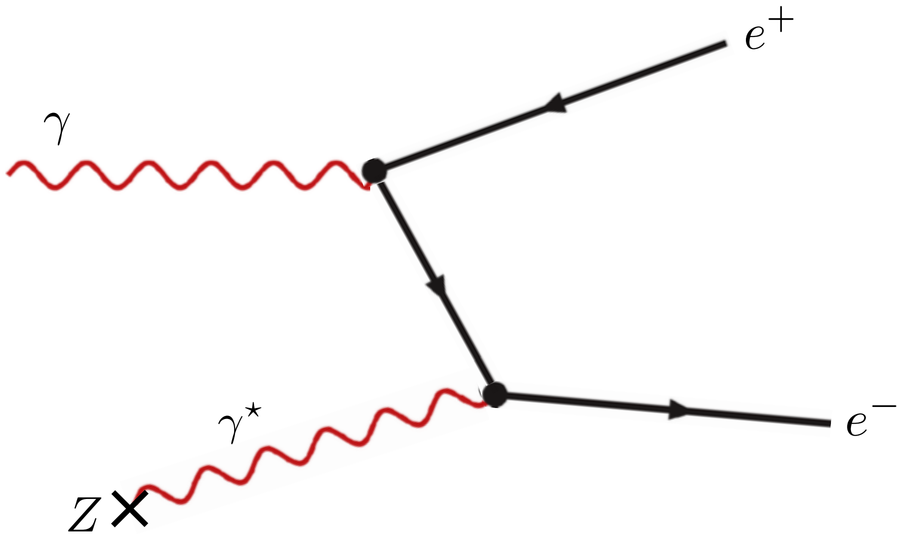
$$s = (\text{center of mass energy})^2 \geq (2m_e)^2$$

$$2E_1E_2(1-\cos\theta_{12}) \geq 4m_e^4$$

- two-vertex process (nucleus electric field)
- virtual photon emitted by atomic nucleus



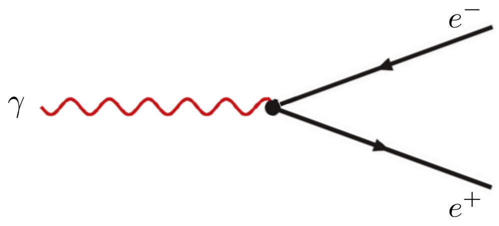
$$\gamma + Z \longrightarrow e^+ + e^- + Z$$



$$\gamma + \gamma^\star \longrightarrow e^+ + e^-$$

e-pair production

electromagnetic shower

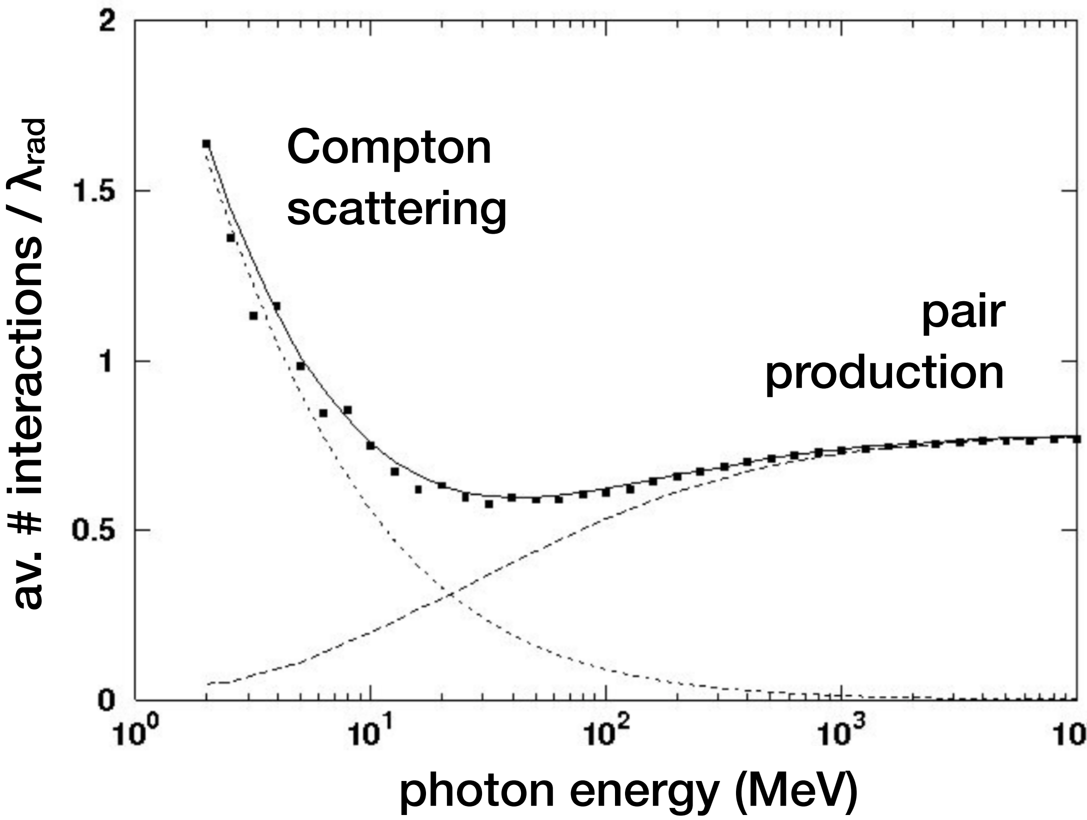


- **$E_\gamma \gg 2m_ec^2$**

- electron-positron pair production
  - threshold process

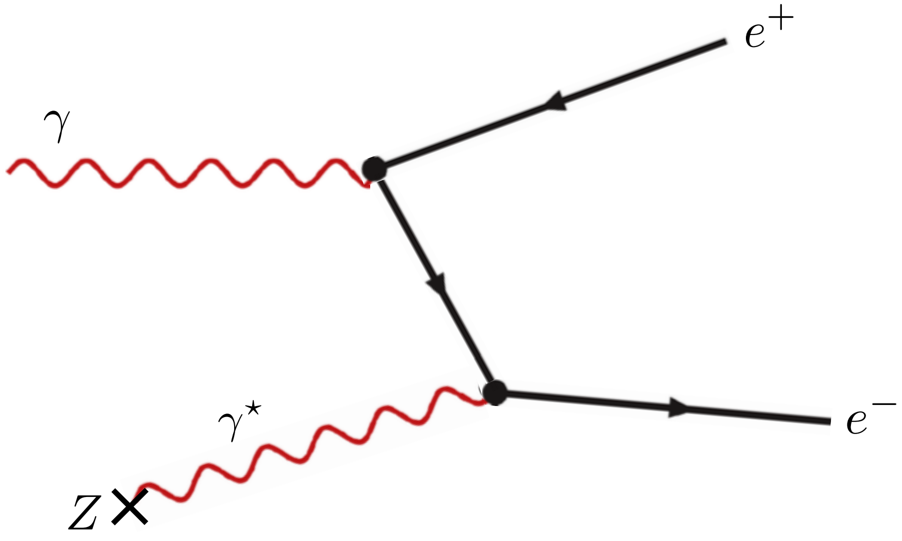
$$s = (\text{center of mass energy})^2 \geq (2m_e)^2$$

$$2E_1E_2(1-\cos\theta_{12}) \geq 4m_e^4$$



- two-vertex process (nucleus electric field)

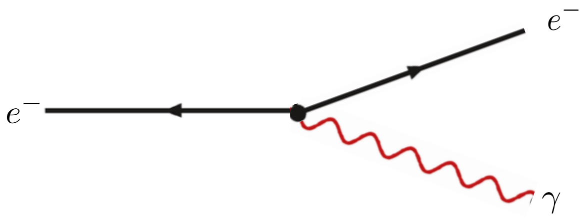
- virtual photon emitted by atomic nucleus



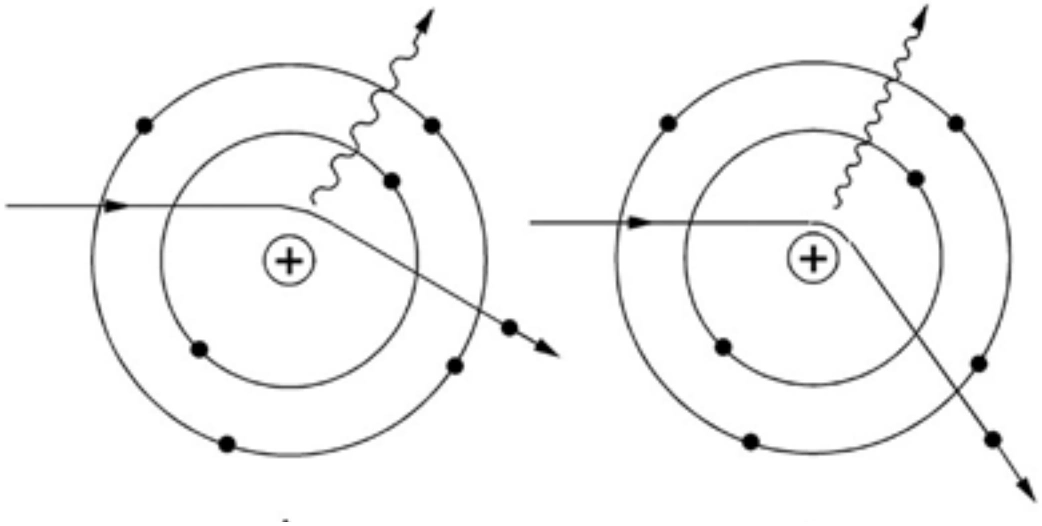
$$\gamma + \gamma^\star \longrightarrow e^+ + e^-$$

bremsstrahlung

electromagnetic shower

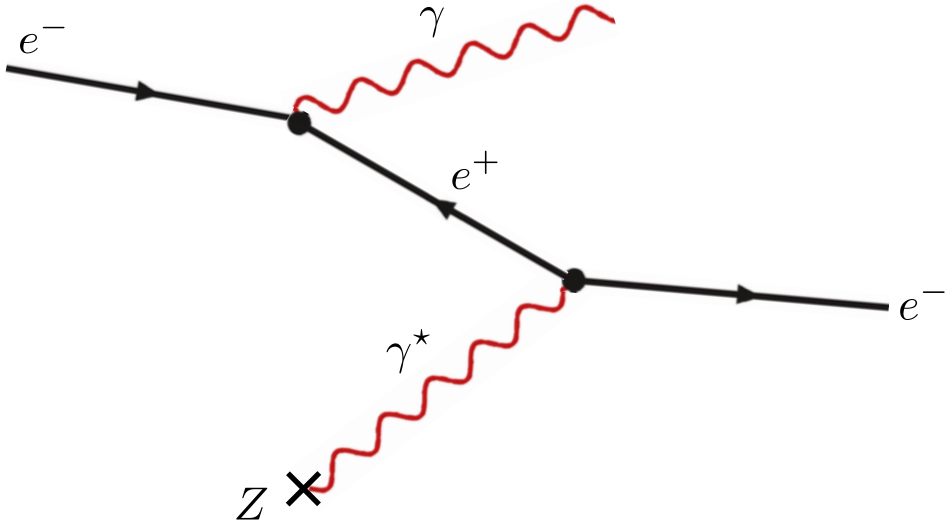


- *breaking radiation*
- EM radiation emitted by deceleration of charge in a Coulomb field



$$e + Z \longrightarrow e + \gamma + Z$$

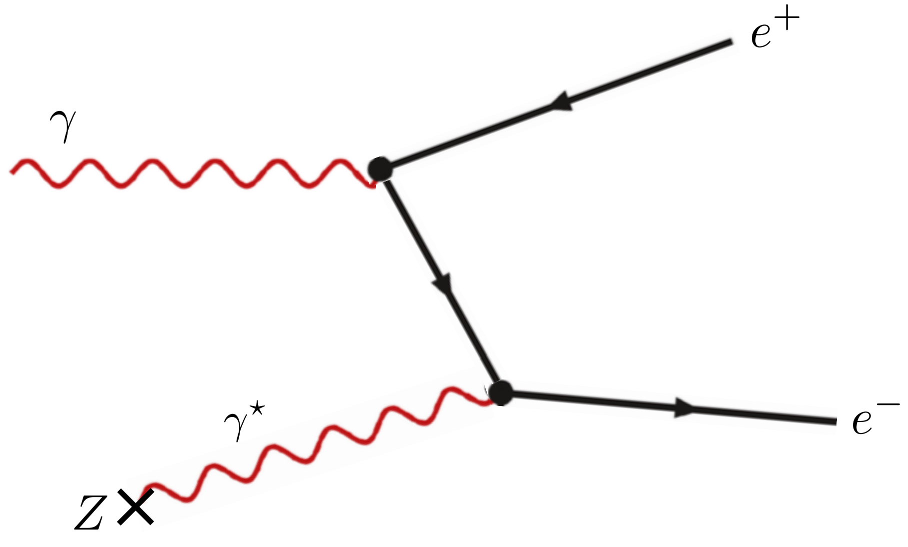
- **e-ion** scattering
- radiation by charge accelerated in nucleus electric field
- astrophysical contribution to X-ray & γ-ray continuum spectra



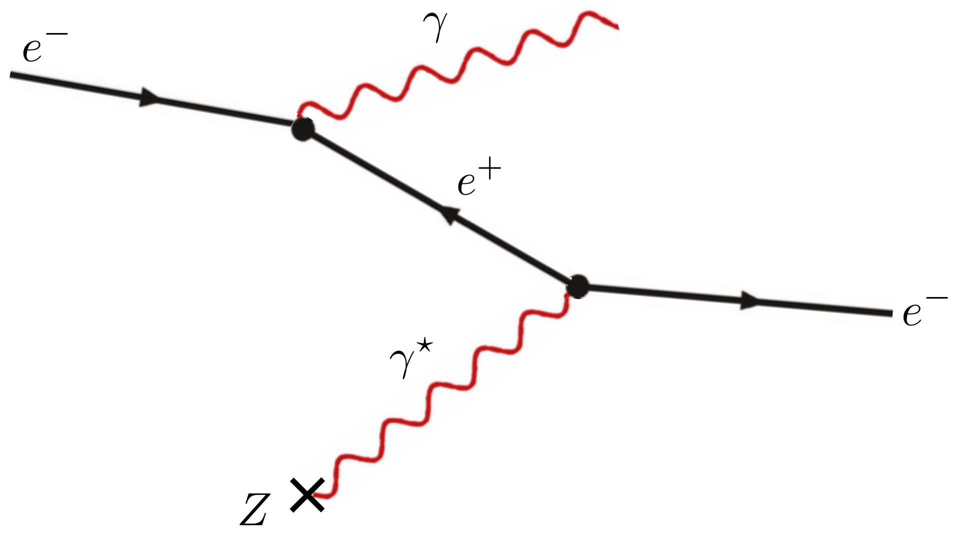
$$e + \gamma^\star \longrightarrow e + \gamma$$

e-pair production & bremsstrahlung

virtual photons



$$\gamma + \gamma^\star \longrightarrow e^+ + e^-$$



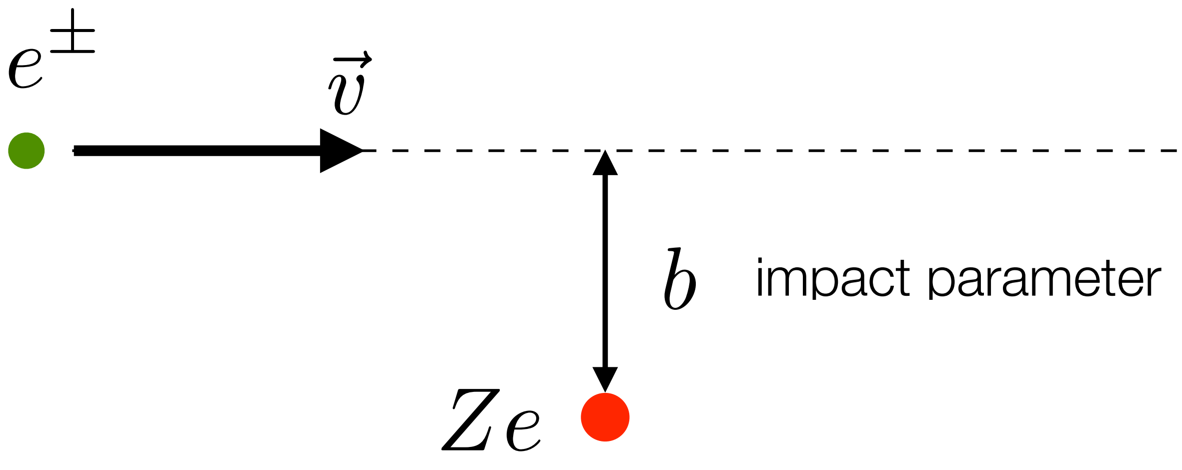
$$e + \gamma^\star \longrightarrow e + \gamma$$

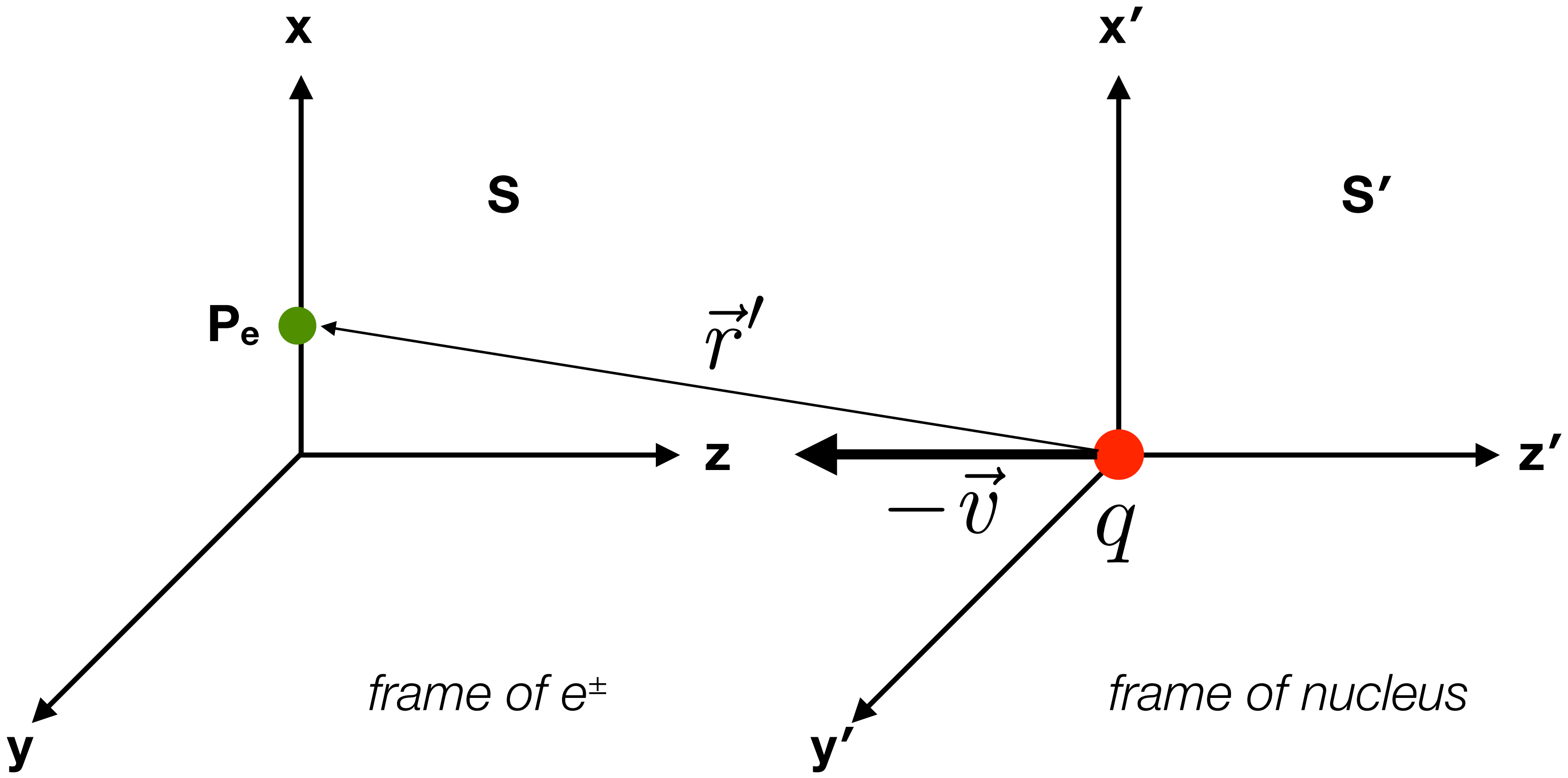
- *similarity between the fields of a rapidly moving charged particle and that of a pulse of radiation*

- bremsstrahlung emission & pair production as *scattering* of **virtual photons** in Coulomb field by the incident particle

method of virtual photons

spectrum of virtual photons





$$\vec{P}_e' = (b, 0, -vt') = (b, 0, -v\gamma t)$$

frame of nucleus

$$\left(\vec{E} = q\frac{\vec{r}}{r^3}\right) \qquad \vec{E}' = \left(\frac{qb}{r'^3}, 0, -\frac{qv\gamma t}{r'^3}\right)$$

$$\vec{B}' = \vec{0}$$

frame of e±

$$\vec{E} = \left(\frac{qb\gamma}{[b^2 + (v\gamma t)^2]^{3/2}}, 0, -\frac{qv\gamma t}{[b^2 + (v\gamma t)^2]^{3/2}}\right)$$

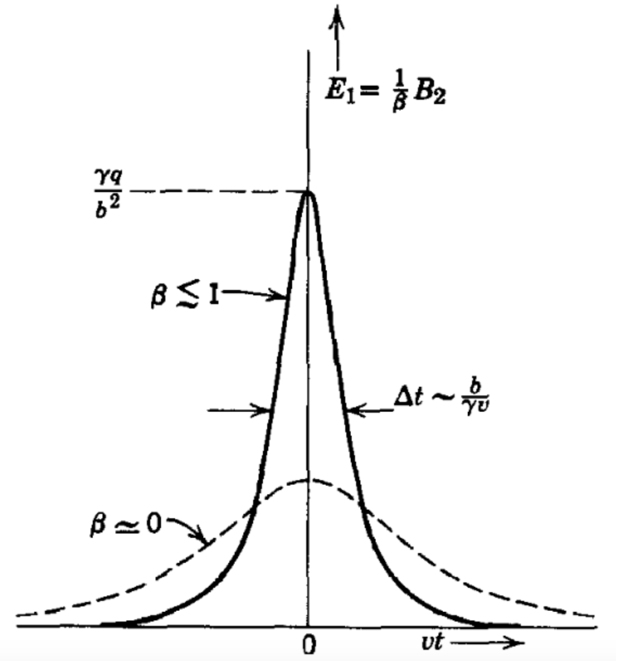
$$\vec{B} = \left(0, \frac{qb\beta\gamma}{[b^2 + (v\gamma t)^2]^{3/2}}, 0\right)$$

method of virtual photons

spectrum of virtual photons

- **electromagnetic pulse** propagating along the z-axis

- $\frac{dI}{d\omega}(\omega, b) = \frac{c}{2\pi} |E_x(\omega)|^2$ energy / unit area (b) / unit frequency interval (ω)



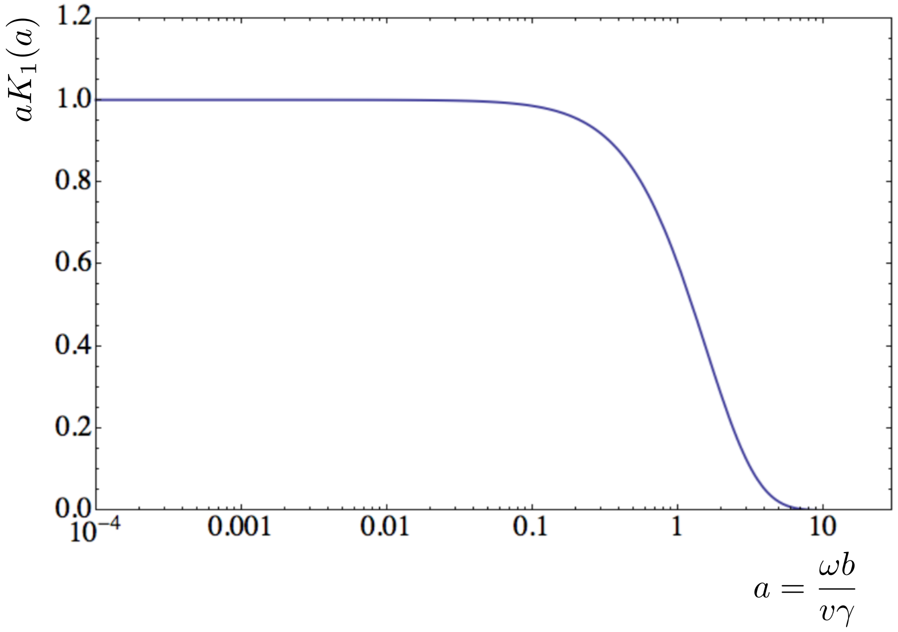
frame of e±

$$\vec{E} = \left(\frac{qb\gamma}{[b^2 + (v\gamma t)^2]^{3/2}}, 0, -\frac{qv\gamma t}{[b^2 + (v\gamma t)^2]^{3/2}} \right)$$

$$\vec{B} = \left(0, \frac{qb\beta\gamma}{[b^2 + (v\gamma t)^2]^{3/2}}, 0 \right)$$

method of virtual photons

spectrum of virtual photons



- **electromagnetic pulse** propagating along the z-axis

- $\frac{dI}{d\omega}(\omega, b) = \frac{c}{2\pi}|E_x(\omega)|^2$ *energy / unit area (b) / unit frequency interval (ω)*

- $E_x(\omega) = \int dt\, e^{i\omega t} E_x(t) = \sqrt{\frac{2}{\pi}} \left(\frac{q}{bv}\right) \left(\frac{\omega b}{v\gamma}\right) K_1\left(\frac{\omega b}{v\gamma}\right)$

- $\boxed{\frac{dI}{d\omega}(\omega, b) = \frac{1}{\pi^2}\frac{1}{b^2}\left(\frac{q^2}{c\beta^2}\right)\left(\frac{\omega b}{v\gamma}\right)^2 K_1^2\left(\frac{\omega b}{v\gamma}\right)}$

$$\approx \frac{q^2}{\pi^2 c\beta^2}\frac{1}{b^2} \qquad \left(\omega \ll \frac{v\gamma}{b}\right)$$

method of virtual photons

spectrum of virtual photons

- $$\frac{dI}{d\omega}(\omega, b) \approx \frac{q^2}{\pi^2 c \beta^2} \frac{1}{b^2}$$

- integrate over impact parameters

= *energy / unit frequency interval (ω)*

- $$\frac{dI}{d\omega}(\omega) = 2\pi \int_{b_{min}}^{b_{max}} \frac{dI}{d\omega}(\omega, b)\, b\, db$$

$$\frac{dI}{d\omega}(\omega) \approx \frac{2q^2}{\pi c \beta^2} \ln\left(\frac{b_{max}}{b_{min}}\right)$$

- number spectrum of **virtual photons**

$$\frac{dI}{d\omega}(\omega) d\omega = \hbar\omega N(\hbar\omega) d(\hbar\omega)$$

$$N(\hbar\omega) = \frac{1}{\hbar^2 \omega} \frac{dI}{d\omega}(\omega)$$

method of virtual photons

spectrum of virtual photons

- number spectrum of **virtual photons**

$$N(\hbar\omega) \approx \frac{2}{\pi\beta^2}\left(\frac{q^2}{\hbar c}\right)\left(\frac{1}{\epsilon_\gamma}\right)\ln\left(\frac{b_{max}}{b_{min}}\right)$$

$(q \equiv Ze)$

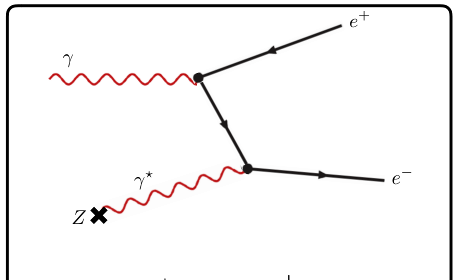
$$N(\hbar\omega) \approx \frac{2\alpha Z^2}{\pi\beta^2}\left(\frac{1}{\epsilon_\gamma}\right)\ln\left(\frac{b_{max}}{b_{min}}\right)$$

- energy of virtual photon

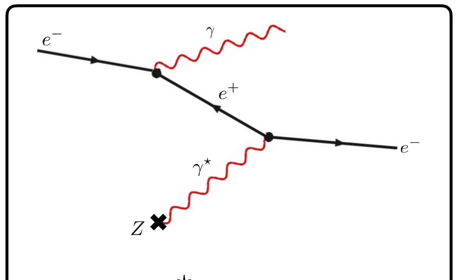
$$\epsilon_\gamma = \hbar\omega$$

method of virtual photons

spectrum of virtual photons



$\gamma + \gamma^\star \longrightarrow e^+ + e^-$



$e + \gamma^\star \longrightarrow e + \gamma$

- bremsstrahlung and pair production as scattering of virtual photons

- virtual photons in nuclear Coulomb field are scattered by the incident particle
- at low frequencies $\left(\omega \ll \frac{v\gamma}{b}\right)$ the Thomson cross section / virtual photon

$$\frac{d\sigma}{d\epsilon_\gamma d\Omega} \approx \frac{r_e^2}{2}(1 + \cos^2\theta) \times N(\hbar\omega)$$

$$\frac{d\sigma}{d\epsilon_\gamma} \approx \alpha Z^2 r_e^2 \frac{1}{\epsilon_\gamma} \ln\left(\frac{b_{max}}{b_{min}}\right)$$

method of virtual photons

spectrum of virtual photons

- b_{min} $\Delta x \Delta p \lesssim \hbar$ $b \lesssim \dfrac{\hbar}{m_e c} \equiv b_{min}$
- b_{max}: for fully ionized nucleus $\dfrac{\epsilon_\gamma b}{\hbar c} \lesssim 1$ $b_{max} \approx \dfrac{\hbar c}{\epsilon_\gamma}$
- : for nucleus in an atom (screening) $b_{max} \lesssim R_{atom}$

full screening approximation

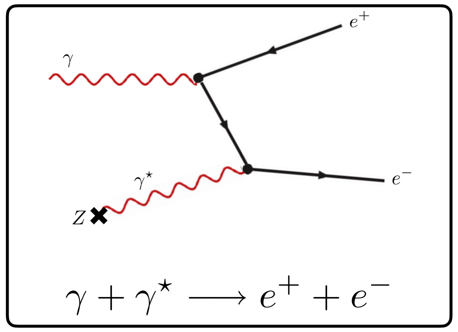
$$R_{atom} \approx \frac{\hbar}{m_e c} 183\, Z^{-1/3}$$

atom of Thomas-Fermi

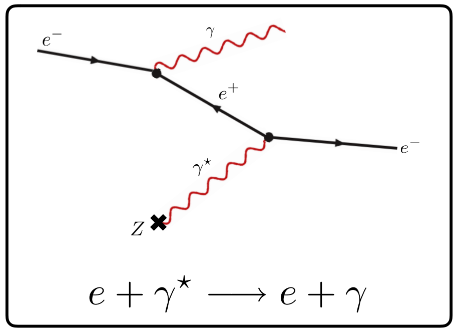
$$R_H \approx a_{Bohr} = \frac{\hbar^2}{m_e e^2}$$

method of virtual photons

spectrum of virtual photons



$\gamma + \gamma^\star \longrightarrow e^+ + e^-$



$e + \gamma^\star \longrightarrow e + \gamma$

- bremsstrahlung and pair production as scattering of virtual photons
- integrate over energy range of virtual photons

$$\frac{d\sigma}{d\epsilon_\gamma} \approx \alpha Z^2 r_e^2 \frac{1}{\epsilon_\gamma} \ln\left(\frac{b_{max}}{b_{min}}\right)$$

- full screening approximation

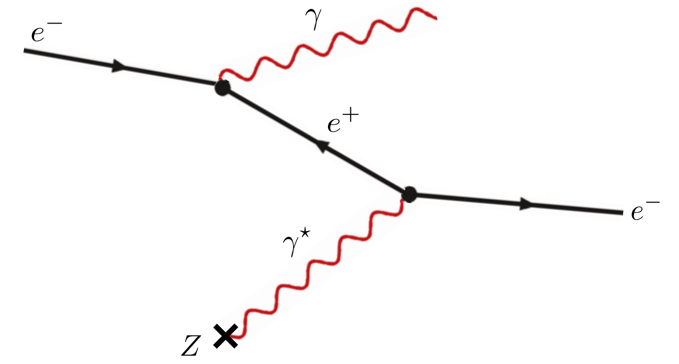
$$\frac{d\sigma}{d\epsilon_\gamma} \approx \alpha Z^2 r_e^2 \frac{1}{\epsilon_\gamma} \ln 183\, Z^{-1/3} \qquad \text{(brems)}$$

e-pair production & bremsstrahlung

electromagnetic showers

$$b \simeq \frac{1}{18 \ln 183 Z^{-1/3}} \qquad b_{air} \simeq 0.0135$$

BREMSSTRAHLUNG

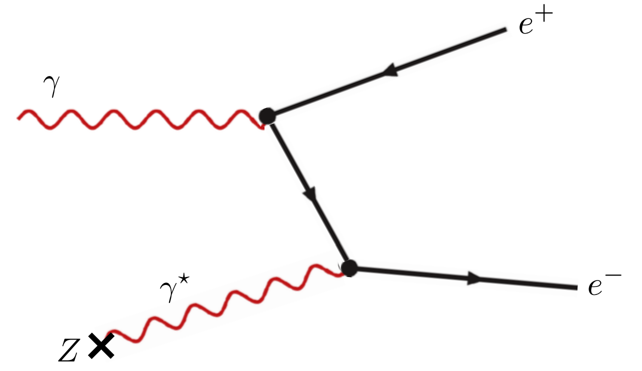


$$e + \gamma^\star \longrightarrow e + \gamma$$

$$\left.\frac{d\sigma}{dv}\right|_{e \to e+\gamma}(v, E_e) = 4Z^2 \alpha r_e^2 \frac{1}{v} \left\{ \left[1 + (1-v)^2 - \frac{2}{3}(1-v) \right] \frac{1}{18b} + \frac{1}{9}(1-v) \right\}$$

$$v = \frac{E_\gamma}{E_e}$$

PAIR PRODUCTION



$$\gamma + \gamma^\star \longrightarrow e^+ + e^-$$

$$\left.\frac{d\sigma}{du}\right|_{\gamma \to e^+ + e^-}(u, E_\gamma) = 4Z^2 \alpha r_e^2 \left\{ \left[u^2 + (1-u)^2 + \frac{2}{3}u(1-u) \right] \frac{1}{18b} - \frac{1}{9}u(1-u) \right\}$$

$$u = \frac{E_{e^+}}{E_\gamma}$$

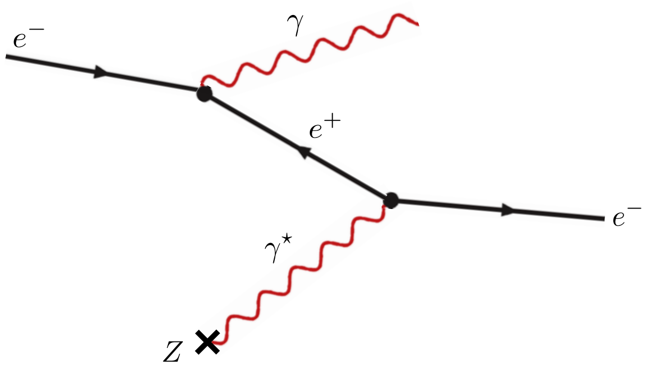
high energy limit
full screening

e-pair production & bremsstrahlung

radiation length

radiation length:
distance where the energy of an electron is reduced to E/e

BREMSSTRAHLUNG



$$e + \gamma^\star \longrightarrow e + \gamma$$

$$\left.\frac{dE_e}{dX}\right|_{brems} = \frac{N_A}{A}\int d\epsilon_\gamma \epsilon_\gamma \frac{d\sigma}{d\epsilon_\gamma}(\epsilon_\gamma, E_e)$$

$$\left.\frac{dE_e}{dX}\right|_{brems} = \frac{N_A}{A} E_e \int_0^1 dv v \frac{d\sigma}{dv}(v) \equiv \frac{E_e}{\lambda_{rad}}$$

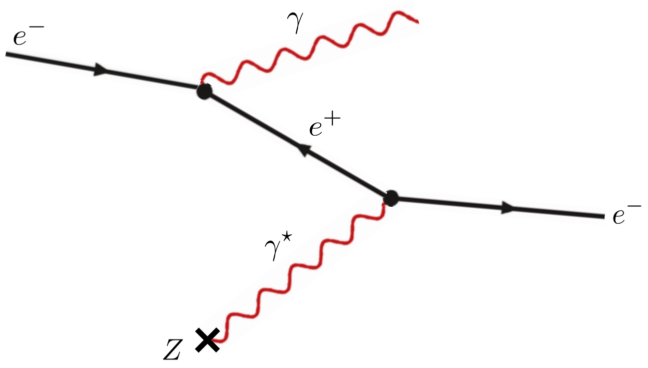
$$\langle E_e(X)\rangle = E_e(0)\, e^{-X/\lambda_{rad}}$$

$$\lambda = \frac{\langle m\rangle}{\sigma}$$

e-pair production & bremsstrahlung

radiation length

BREMSSTRAHLUNG



$e + \gamma^\star \longrightarrow e + \gamma$

radiation length:
distance where the energy of an electron is reduced to E/e

$$\left.\frac{dE_e}{dX}\right|_{brems} = \frac{N_A}{A} E_e \int_0^1 dv v \frac{d\sigma}{dv}(v) \equiv \frac{E_e}{\lambda_{rad}}$$

$$\boxed{\frac{1}{\lambda_{rad}} = \frac{N_A}{A} 4Z^2 \alpha r_e^2 \left[\ln 183 Z^{-1/3} + \frac{1}{18}\right]}$$

radiation length

$$\boxed{\frac{1}{\lambda_{rad}} = \frac{N_A}{A} 4Z^2 \alpha r_e^2 \left[\ln 183 Z^{-1/3} + \frac{1}{18} \right]}$$

$$\frac{1}{\lambda_{rad}} = \frac{N_A}{A} 4Z(Z+1) \alpha r_e^2 \ \ln 183 Z^{-1/3}$$

$$\frac{1}{\lambda_{rad}} = \frac{N_A}{A} 4\alpha r_e^2 \ \{ Z^2 [L_{rad} - f(Z)] + Z L'_{rad} \}$$

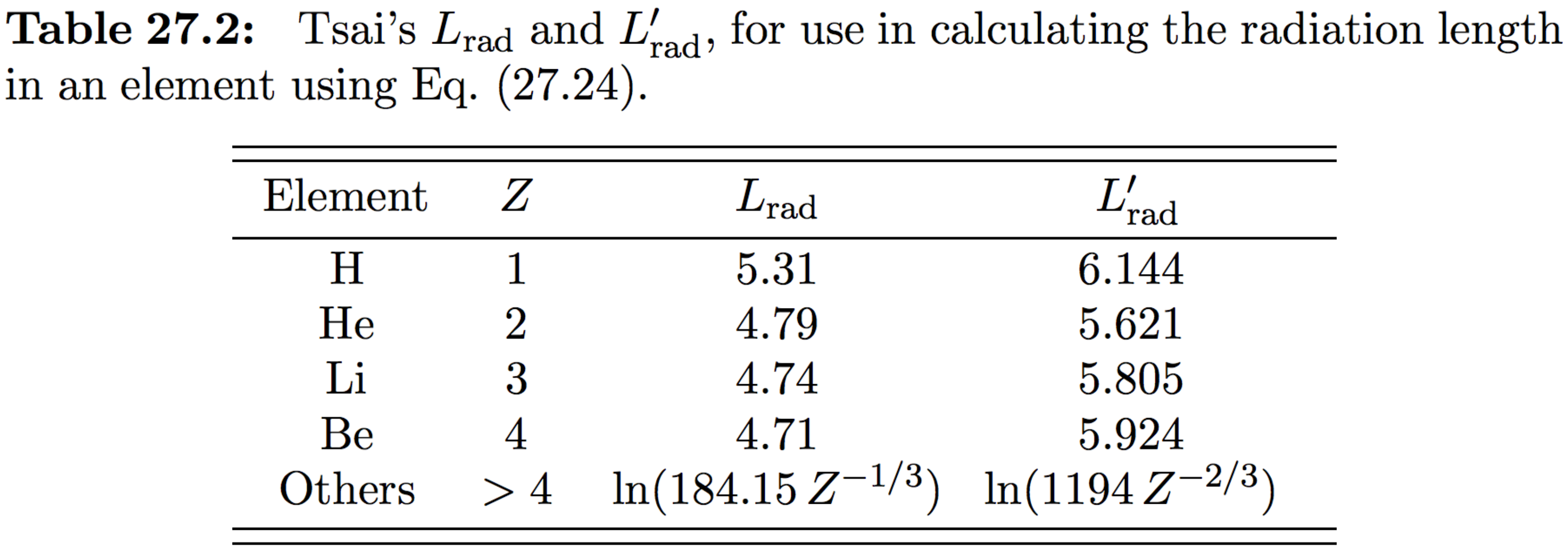


Table 27.2: Tsai's $L_{\rm rad}$ and $L'_{\rm rad}$, for use in calculating the radiation length in an element using Eq. (27.24).

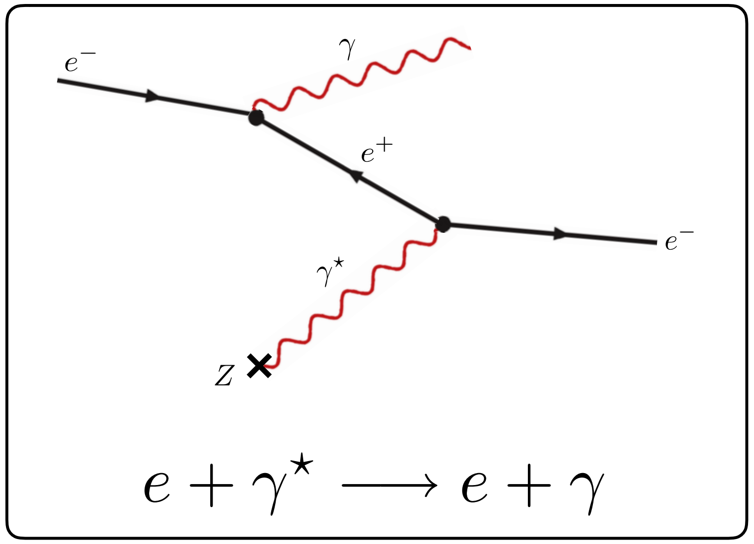
Element	Z	$L_{\rm rad}$	$L'_{\rm rad}$
H	1	5.31	6.144
He	2	4.79	5.621
Li	3	4.74	5.805
Be	4	4.71	5.924
Others	> 4	$\ln(184.15\, Z^{-1/3})$	$\ln(1194\, Z^{-2/3})$

PDG

e-pair production & bremsstrahlung

splitting function

BREMSSTRAHLUNG



$$e + \gamma^\star \longrightarrow e + \gamma$$

radiation length:
distance where the energy of an electron is reduced to E/e

$$\varphi(v) = \frac{d\sigma}{dv}(v)\bigg|_{brems} \left(\frac{N_a}{A}\lambda_{rad}\right)$$

$$\boxed{\varphi(v) = \frac{1}{v}\left[1 - \left(\frac{2}{3} - 2b\right)(1-v) + (1-v)^2\right]}$$

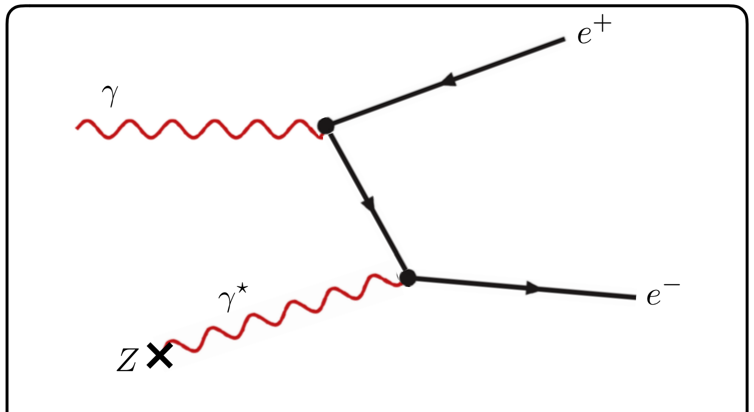
probability per unit of λ_{rad} that an $e^\pm$ of energy E_e emits a photon of energy $E_\gamma = vE_e$

$$\int_0^1 dv\, v\, \varphi(v) = 1 + b$$

e-pair production & bremsstrahlung

photon mean free path

PAIR PRODUCTION



$$\gamma + \gamma^\star \longrightarrow e^+ + e^-$$

photon mean free path:
9/7 of the radiation length

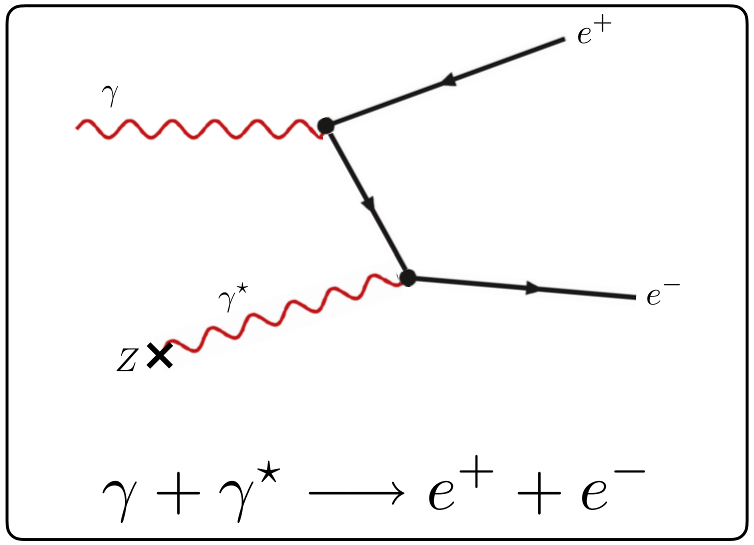
$$\left.\frac{dE_\gamma}{dX}\right|_{epair} = \frac{N_A}{A} E_\gamma \int_0^1 du u \frac{d\sigma}{du}(u) \equiv \frac{E_\gamma}{\lambda_{pair}}$$

$$N_\gamma(X) = N_\gamma(0)\, e^{-X/\lambda_{pair}}$$

e-pair production & bremsstrahlung

photon mean free path

PAIR PRODUCTION



$$\gamma + \gamma^\star \longrightarrow e^+ + e^-$$

photon mean free path:
9/7 of the radiation length

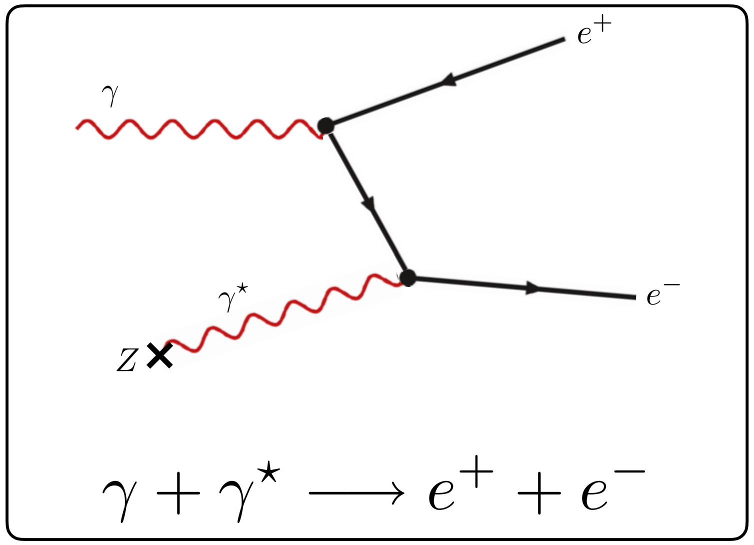
$$\left.\frac{dE_\gamma}{dX}\right|_{epair} = \frac{N_A}{A} E_\gamma \int_0^1 duu \frac{d\sigma}{du}(u) \equiv \frac{E_\gamma}{\lambda_{pair}}$$

$$\boxed{\lambda_{pair} \simeq \frac{9}{7}\lambda_{rad}}$$

e-pair production & bremsstrahlung

photon mean free path

PAIR PRODUCTION



$$\gamma + \gamma^\star \longrightarrow e^+ + e^-$$

photon mean free path:
9/7 of the radiation length

$$\psi(u) = \frac{d\sigma}{du}(u)\Big|_{epair}\left(\frac{N_A}{A}\lambda_{rad}\right)$$

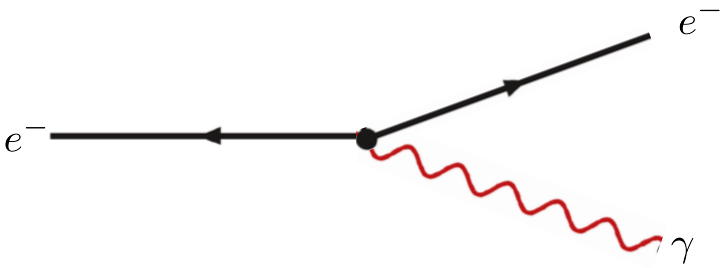
$$\boxed{\psi(u) = (1-u)^2 + \left(\frac{2}{3} - 2b\right)(1-u)u + u^2}$$

$$\sigma_0 = \int_0^1 du\, \psi(u) = \frac{7}{9} - \frac{b}{3}$$

probability per unit of λ_{rad} that a γ of energy E_γ produces a pair with e^+/e^- of energy
$E_e = uE_\gamma$

electromagnetic showers

BREMSSTRAHLUNG



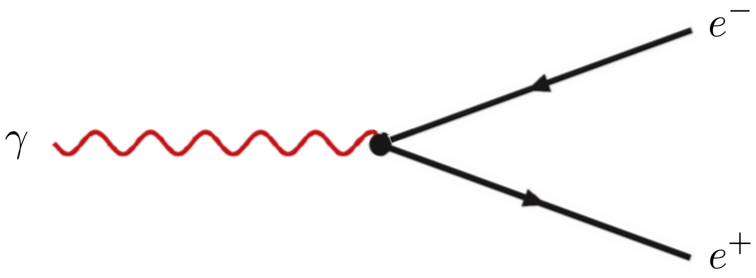
$$\varphi(v) = \frac{1}{v}\left[1 - \left(\frac{2}{3} - 2b\right)(1-v) + (1-v)^2\right]$$

$$\frac{1}{\lambda_{rad}} = \frac{N_A}{A} 4Z^2 \alpha r_e^2 \ln 183 Z^{-1/3}$$

$$\lambda_{rad}^{\text{air}} \simeq 37\,\text{g/cm}^2$$

probability per unit of λ_{rad} that an $e^\pm$ of energy E_e emits a photon of energy $E_\gamma = vE_e$

PAIR PRODUCTION



$$\psi(u) = (1-u)^2 + \left(\frac{2}{3} - 2b\right)(1-u)u + u^2$$

$$\lambda_{pair}^{\text{air}} \simeq 47\,\text{g/cm}^2$$

probability per unit of λ_{rad} that a γ of energy E_γ produces a pair with e^+/e^- of energy $E_e = uE_\gamma$

THANK YOU

***workshop* in particle physics**

development of electromagnetic showers

references

- **printed material**
  - *Classical Electrodynamics* - JD Jackson - Chapter 15

  - Cosmic Rays and Particle Physics. Thomas K. Gaisser