EXERCISE SHEET I

Exercise 1:

Analyse the properties of different astrophysical systems with given electron density n_e, temperature T_e and size L of the considered systems:

System	**Interst. gas**	**Mol. cloud**	**Sol. corona**	**AGN**	**Cluster of galaxies**	**Cosmic rays**
$n_e[\mathrm{cm}^{-3}]$	0.1	1	10^6	10^9	10^{-2}	10^{-9}
$T_e[\mathrm{K}]$	10^4	10^2	10^6	10^7	10^7	10^{12}
$L[\mathrm{cm}]$	10^{18}	10^{17}	10^{10}	10^{15}	10^{23}	10^{18}

(a) Calculate the electron plasma frequency ω_{pe} and the Debye length λ_D of the different astrophysical systems.

(b) For which systems is a description as a plasma appropriate?

(c) Use the Table to calculate the electron-electron collision length $\lambda_{ee} = v/\nu_{ee}$. Using the cross-section $\sigma = \pi r^2$ in the case of two colliding particles with relative speed $\bar{v}$ and effective mass $\bar{m}$, the collision frequency is given by $\nu_{ee} = 4\pi e^4 n v/(\bar{m}^2\bar{v}^4)$. Here you can assume that $v \approx \bar{v} = v_{th}$ and $\bar{m} = m_e$.

Are elastic 2-body collisions between cosmic ray electrons important?

(d) In a plasma the electrostatic interaction of the particles has to dominate over the usual kinetic of the gas. Does this criterion lead to a third requirement for a plasma we haven't discussed yet?

Exercise 2:

Determine the change of elements in our Milky Way due to the effect of spallation. Consider an instantaneous injection of medium heavy elements n_M (C, N, O) which decay into light elements n_L (Li, Be, B) with a probability p_{LM}.

a.) Determine the differential equations for n_L and n_M by using the leaky box model (neglect momentum changes).

b.) Calculate the temporal development of n_L and n_M and show that

$$\begin{aligned} n_M(t) &= n_{m,0} \exp\left(-\frac{t}{\tau_M}\right) \\ n_L(t) &= \frac{p_{LM} n_M(t) \tau_L}{\tau_M - \tau_L}\left[1 - \exp\left(\frac{t}{\tau_M} - \frac{t}{\tau_L}\right)\right] \end{aligned} \tag{1}$$